CRITICAL STUDY

Newton C.A. da Costa, *O Conhecimento Científico.* (São Paulo, Discurso Editorial, 1997), 278 pp. ISBN 85-86590-01-0.

A NATURALISTIC REVIEW OF A TREATISE ON THE LOGIC OF SCIENTIFIC KNOWLEDGE

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1. PREAMBLE

When Niels Bohr proposed his quantum model of one-electron atoms, in 1913, he did not only give a successful explanation for a class of spectroscopic data, but also made certain experimental predictions that were soon confirmed. In Vienna, his Hungarian friend Georg von Hevesy informed Albert Einstein of these results, and the latter's reaction is registered, in slightly eccentric English, in a letter of Hevesy's to Bohr:

> [...] When he heard this he was extremely astonished and told me: "Then the frequency of the light does not depend at all on the frequency of the electron" [...] And this is an *enormous achiewement.* The theory of Bohr must then be wright [...] (Mehra & Rechenberg (1982), p. 201).

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Did he consider Bohr's theory to be *true*? Certainly not as a full-fledged theory (once he got acquainted with its contradictory foundations), but in the restricted domain of one-electron atoms it seemed "right", it saved the appearances.

What most struck Einstein was the statement inside the quotation marks, which amounts to a falsification of a hypothesis adopted in the wake of Lorentz's theory (1892) of the electron. Is the statement *true*? Apparently, yes: any future theory would have to incorporate this discovery.

So what about the truth of Bohr's atomic theory? The theory contained true theoretical statements (such as the one inside quotation marks) and generated some true observational statements (predictions concerning the spectrum of the helium ion). But it failed outside the domain of one-electron atoms. It is clearly not a true theory *in totum*, although one could say that it is partially true or approximately true. And even nowadays it may be used to obtain accurate predictions in its restricted domain, while some of its concepts (such as the discrete orbits) are extrapolated to other domains as a "semi-classical approximation".

2. OVERVIEW OF THE BOOK

What is the place in science of partially true theories, such as Bohr's? The renowned Brazilian logician Newton da Costa, in his latest book *Scientific Knowledge* (published in Portuguese), argues that such theories will always be partially true and therefore potentially useful. His central thesis is that science is a set of interrelated and sometimes inconsistent cognitive systems that strives not for truth, in the sense of the correspondence theory of truth (which the author nonetheless endorses), but for truth in the pragmatic sense, one which only "saves the phenomena". The core of his approach is a logically precise definition of such partial truth, which we will examine and attempt to criticize.

The book is composed of five chapters, which are divided into several untitled sections. This first edition published by Discurso Editorial (based at the Department of Philosophy at the University of São Paulo) has not been very well prepared, containing many typographical errors, lacking a decent table of contents and offering no index whatsoever. A second revised edition has just been relesead, and a Spanish edition is currently being prepared in Mexico, at UNAM.

The reviewer's aim is to convey to the reader a summary of the most interesting theses put forth by this highly creative and original thinker, drawing a picture of his philosophical worldview. I will also do my best to criticize the book's shortcomings, in an attempt to generate a debate that will strengthen one of the few original traditions in Brazilian philosophy of science.[2]

The reviewer would also like to stress that he is approaching the book from the specific perspective allowed by his academic back-

[2] The history of Brazilian philosophy of science may be characterized as a set of studies (disconnected from each other) of the theories of science developed in Europe and North America. There is practically no tradition in the field, in contrast to the situation in the natural sciences, and even in the field of logic. In the 1980's, a promising program, which included the history and philosophy of science, was established at the State University of Campinas (UNICAMP), only to be thwarted by the older philosophical tradition of doing research almost exclusively in history of philosophy. Presently there are a few centers which are growing slowly, some of which stimulated by the experience in Campinas. Besides the University of São Paulo (USP), where da Costa works, we might mention the groups at the Federal Universities in Brasília (UnB), Florianópolis (UFSC), Porto Alegre (UFRGS), and Rio de Janeiro (UFRJ). Da Costa is basically a logician, but he has surrounded himself with an excellent group of young researchers (most of whom have contributed to the book being reviewed) at the Philosophy Department of the University of São Paulo (USP), and seems to have established a durable indigenous tradition in the philosophy of science and mathematics.

ground.[3] Many of the other potential reviewers, with a better knowledge of logic, foundations of mathematics, and the structural theory of partial truth, turned out to be contributers to da Costa's treatise, in the form of short appendices distributed throughout the volume.[4]

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[3] The reviewer has studied the concepts and history of quantum physics. The perspective adopted in this review may be characterized as "naturalistic", accepting the results of science as relevant for philosophical investigation. Concerning the metatheories of scientific development, the reviewer tends to distrust excessive use of logic. He believes that the bottleneck deterring progress in the field is the lack of an adequate systematization of the great amount of historical information against which the different metatheoretical descriptions may be tested. This problem can only be overcome with the aid of computers.

[4] I will now briefly describe some of these short articles, giving some information about da Costa's collaborators. The Preface of the book is written by Jean-Yves Béziau, a French logician working at the National Laboratory for Scientific Computation (LNCC) in Petrópolis. He also contributes appendices on universal logic and valuation theory. Stephen French, the well-known philosopher of science from the University of Leeds, discusses what it means to be rational within the context of the theory of partial truth. His doctoral student, Otávio Bueno, now at California State University, Fresno, writes three nice pieces on the history of paraconsistent logic, on inductivism, and on the formal theory of learning, the last in collaboration with Antônio Mariano Coelho, Professor of Logic at the Federal University of Santa Catarina in Florianópolis. Edélcio de Souza, from the Catholic University in São Paulo (PUC-SP), writes about multideductive logics. From the state of Paraná, where da Costa started his career, Décio Krause (Federal University of Paraná, in Curitiba) summarizes his own pioneering work on the theory of quasi-sets and its applications for describing quantum non-individuality. The mathematical physicist Francisco Antônio Dória (Federal University of Rio de Janeiro), who forms with da Costa the Lennon & McCartney duet of Brazilian logic of physics, describes the history of their successful program for axiomatizing parts of the empirical sciences, which include physics and the social sciences, and the undecidability results obtained. The undecidability involved in the attempt to unite Lotka's dynamics with the mathematical theory of economical equilibrium of Arrow & Debreu is further developed in an appendix by the economist Marcelo Tsuji, which follows another appendix on the axiomatization of economic science

This limitation of the reviewer, I hope, will be compensated by a more picturesque and easier to understand (for those who don't manipulate symbols in their daily work) description of the logical theory of partial truth.

Chapter I: Science and Knowledge

Sec. 1, 9, 15-16: Introduction to knowledge and rationality in science.
Sec. 2-8, 17, notes I-III: Survey of the standard definition of knowledge.
Sec. 10-14: Discussion of the classification of the areas of knowledge.
Sec. 18-26: Overview of five traditional problems in the theory of knowledge.
Sec. 27-29: Logical underpinnings of scientific theories.

Chapter II: The Formal Sciences

Sec. 1-5: Evolution of mathematics and discussion of its foundations.
Sec. 6-10: Survey of the field of logic.
Sec. 11-14: General theses concerning the relation between logic and maths.

Chapter III: Truth

Sec. 1-2, 8, 14: Three traditional conceptions of truth.
Sec. 3-7: Introduction to Tarski's theory of truth.
Sec. 9-12: Theory of partial truth.
Sec. 13,15: Additional issues of the theory of pragmatic truth.

Chapter IV: The Empirical Sciences

Sec. 1-4: Scientific pluralism and the use of incompatible theories.
Sec. 5-6: The Semantical approach and theory of partial truth.
Sec. 7-12: Pragmatic acceptance of theories *via* inductive logic.

Chapter V: Scientific Rationality

Sec. 1-2: Overview of rationality in science and scientific pluralism.
Sec. 3-7: Methodology of science: further discussions.
Sec. 8-11: The relations between logic and physics.

Table 1: *Main topics of* O Conhecimento Científico *by Newton da Costa.*

written by Roque Caiero, both from the University of São Paulo (USP). Finally, Nelson Papavero, a zoologist retired from USP and now at the National Autonomous University of Mexico (UNAM), summarizes the field of axiomatization of the theories of comparative biology. At the end of the book, there is a nice interview with da Costa, conducted by Caetano Plastino, from the Philosophy Department at USP.

Da Costa's book might be profitably read by starting from the topic that most interests the reader. One interesting route would be to start with Chapter II, then proceding to III, IV, I, V. To guide the reader, a brief summary of the topics covered in the book is presented in Table 1.

A summary of the topics covered in this review is given below, inviting the reader once again to start reading from the point that most attracts him:

§3. *General Theses about Knowledge.* Some of da Costa's general views on knowledge in the empirical and formal sciences.

§4. *Some Remarks on the Formal Sciences.* The evolution of mathematics and its foundations; survey of different logical systems.

§5. *Truth.* Three traditional conceptions of truth; introduction to Tarski's definition of truth (refer to Fig. 1).

§6. *Pragmatism and Partial Truth.* Brief examination of pragmatism; the intuitive idea of partial truth; da Costa's antirealism.

§7. *The Semantical Approach to Theories.* Semantical approach to the axiomatization of scientific theories (refer to Fig. 2).

§8. *The Logic of Partial Truth.* Da Costa *et al.*'s theory of partial (or pragmatic) truth (refer to Fig. 3); discussion of the relevance of the theory.

§9. *Scientific Development.* Approximate truth; cumulative view of scientific development; critique of Popperian methodology; criterion of demarcation; the Duhem thesis.

§10. *Theory of Knowledge and the Principles of Science.* The definition of knowledge as justified true belief; five traditional problems in the theory of knowledge; general principles defining the aims of science; three metaphysical postulates.

§11. *Pragmatic Probability.* Extended inductivism; qualitative, comparative, and metric probabilities as measures of degree of rational belief.

§12. *Incompatibilities in the Empirical Sciences.* Scientific pluralism and inconsistencies within and between theories.

§13. *Logic and Physics.* Gödel's incompleteness theorem applied to chaos theory; the Sitnikov-Alekseev theorem; quasi-sets and quantum indistinguishability; criticism of the argument that the unique underlying logic of quantum physics is non-distributive.

3. GENERAL THESES ABOUT KNOWLEDGE

We will start by highlighting some general theses concerning knowledge in the empirical and formal sciences, presented by da Costa in his first two chapters, which mark his overall pragmatic approach to knowledge.

(1) *Knowledge.* Da Costa starts out by accepting the standard definition of knowledge as "justified true belief": someone *knows* a proposition *p* if and only if he *believes* that *p*, *p* is *true*, and the belief in *p* is *justified.* In the formal sciences, knowledge involves truth as correspondence and justification as intuitive evidence (p. 95). In the empirical sciences, da Costa concludes that knowledge is "belief that is pragmatically true and justified with high pragmatic probability".

(2) *Truth.* The basic definition of truth adopted by the author is given by Tarski's correspondence theory. Da Costa's proposal for a theory of "partial truth" (which in the book is synomymous with "pragmatic truth"[5] and "quasi-truth") attacks the

[5] In spite of using the term "pragmatic truth" throughout the book, in conversation da Costa has said that he tends not to use the term in order to avoid confusion with the pragmatism of Peirce and James. For him, it is

problem of incomplete knowledge within the framework of such a correspondence theory. However, in line with his pluralistic attitude, he allows that other definitions of truth, especially the coherence theory, may be adopted in science and mathematics.

(3) *Rationality.* Four dimensions characterize rationality in science (pp. 35-7): the existence of concepts (which the author does not define explicitly), of deduction (usually according to classical logic), of non-deductive inferences (comprising induction, analogy, statistical inference, and the hypothetico-deductive method), and of a critical attitude.

(4) *Intuition.* Knowledge is basically rational, but there is a fundamental place for intuition. In a mathematical proof, for example, passing from one step of proof to the next involves intuitive knowledge (p. 47). In the empirical sciences, da Costa is not clear about whether intuition has a place only in the context of discovery.

(5) *Scientific pluralism.* There is nothing preventing science from being constituted by different, appropriately interconnected, cognitive or conceptual systems (pp. 29, 91). The inspiration for this quasi-Feyerabendian thesis comes from the situation in logic, where (at the metatheoretical level) classical logic coordinates the investigation of the most diverse systems of logic. In this respect, da Costa's motto could be: "Anything goes, as long as rationality is maintained".

(6) *Underlying logic.* Any cognitive system contains an underlying logic, which furnishes the rules of deduction and inference within the cognitive system (pp. 13, 91). Such a logic may be different from classical logic. Logic precedes mathematics in the

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an open problem of exegesis to compare his approach with that of the American pragmatists.

sense that mathematics requires proof, and the basis of proof is logic.

(7) *Intuitionistic metamathematics.* Any abstract conceptual system, a mathematical theory, has a formal linguistic dimension and a metalinguistic dimension, which is informal and intuitive. As such, the most appropriate foundation for metamathematics is intuitionistic logic rather than classical logic (p. 96, but compare p. 140). This conclusion might also be extended to the natural sciences (p. 203).

(8) *Cumulative conception of science.* Science is essentially cumulative, even during scientific revolutions, and diverse paradigms are in principle commensurable. Da Costa's theory of partial truth will support the thesis that the development of science is cumulative in a direct way.

4. SOME REMARKS ON THE FORMAL SCIENCES

Chapter I, entitled "Science and Knowledge", is an uneven introductory chapter, raising some important issues about knowledge in the empirical sciences, but not flowing in a pleasant way for the reader. The author presents a survey of the theory of knowledge which is schematic and not very original (see §10). The main interest of this chapter for future generations, apart from some seminal ideas concerning epistemic logic and the principles of science, is probably what it reveals about da Costa's world view.

In contrast to the uneven beginning, Chapter II, "The Formal Sciences", is a masterpiece of popularization of mathematics and logic. In thirty pages, which can be read independently of the previous chapter, da Costa presents two centuries of the most important achievements in the foundations of mathematics and logic. What is especially nice about this historical survey is that the author combines great ability for summarizing and systematizing the important advances with a deep knowledge of many details, which he offers in a casual style to the reader.

The first five sections cover the evolution of mathematics and the discussion of its foundations. Some of the most interesting assertions, for someone ignorant of the field, such as the reviewer, are:

(a) Correct mathematical statements are *conditonal,* not categorical: the truth of a statement ("the sum of the angles of a triangle are equal to two right angles") depends on the mathematical theory being considered (for example, hyperbolic geometry).

(b) *Non-Cantorian mathematics*, based on set theories in which the axiom of choice or the continuum hypothesis are not valid, have been employed in physics. One may speculate that one such mathematical theory might turn out to be advantageous in certain areas of mathematical physics.

(c) *Category theory* can only be founded in set theory if the existence of additional sets is postulated, and can only be encompassed within the concept of "structure" (proposed by Bourbaki, the pseudonym of the famous group of French mathematicians) if their modifications are introduced.

(d) *Foundational pluralism.* There is no unique and definite foundation for mathematics: one may employ set theory, category theory, or even the theory of types.

The author then presents a survey the field of logic, understood both as part of pure mathematics and as an applied discipline, the theory of valid inference. Classical logic has been extended in different directions, forming complementary logics such as modal, temporal, deontic, and infinitary logics. Da Costa, however, focuses especially on strictly non-classical or heterodox logics (those that deny some of the basic axioms of classical logic), which he divides into four main groups:

(i) Paraconsistent logics allow inconsistencies ("A" and "not-A" might both be true sentences) without resulting in a trivial theory (in which every sentence is deducible). Their semantics violates the Aristotelian law of contradiction.

(ii) Paracomplete logics exclude, in their semantics, the law of the excluded middle. They encompass multivalued logics, fuzzy logic, and intuitionist systems such as the Brouwer-Heyting logic.

(iii) Non-alethic logics are both paraconsistent and paracomplete.

(iv) Non-reflexive logics violate some form of the law of identity, such as $\forall x\ (x = x)$.

As is well known, da Costa's first important work was his pioneering 1963 article furnishing a system of paraconsistent logic (for a historical overview of the development of the field, see d'Ottaviano, (1990)), and since then he has published extensively on the subject. Yet, he does not delve into paraconsistent logic in the present book, and in these sections on strictly non-classical logics he gives more emphasis to intuitionistic logic and to a system of non-reflexive logic that is applicable to indistinguishable particles in physics (we will comment on this system in § 13).

5. TRUTH

While Chapter II skims gracefully over the surface of the formal sciences, Chapter III dives deeply into the intricacies of the author's theory of partial truth.

The chapter starts out with an introduction to the correspondence theory of truth and to Tarski's theory of truth. The discussion is authoritative, clear, rich, and interesting, although it is not the most didactical introduction to the subject one may find in the literature, mainly because of its brevity.[6] The reviewer, for example, was quite at lost until he read through Haack ((1978), ch. 7) and Kirkham (1992), while Tarski (1944) is also quite readable.

Da Costa starts out by defending that the aim of science is to attain the truth, and mentions the three traditional *conceptions* of truth: correspondence, pragmatist, and coherence. The first conception states that the truth of a proposition is given by its "correspondence" to the facts. The coherence conception (section 14) claims that truth is a relation of coherence between a statement and a system of beliefs (not a relation between a statement and the facts). The pragmatist theory (section 8), putting it roughly, stresses that the truth of an assertion depends on the practical consequences of the acceptance of the assertion.

In spite of the existence of these three grand schools (and many later offsprings), the emphasis given by the author to the classical (Aristotelian) correspondence conception suggests that this conception is somehow privileged. Da Costa is not explicit

[6] Da Costa is well known for giving exciting addresses to general audiences, attracting excellent students to his ranks, and for conducting superb seminars for advanced students in logic, and being a very good thesis advisor. However, he is not the best teacher for introductory courses in logic, since he lacks the patience of going through all the trivial details with the students. The book being reviewed reflects this profile. At times it is very stimulating for the lay reader, but then, suddenly, he presents several pages with the details of a logical system which the average reader cannot follow, unless he knows his logic well. At times he refers to physical concepts that a logician ignores, and then to logical concepts that a scientist ignores. In spite of its didactical deficiencies, da Costa's book is a veritable goldmine of fruitful ideas.

about exactly why this is so, but we may draw on a distinction first made by Russell (1908) between "definitions" of truth and "criteria" of truth: while the *definition* gives the meaning of "truth", a *criterion* furnishes a test to decide whether a sentence is true or false (Haack, (1978), p. 88). The reviewer could only put some order to his ideas after he adopted Russell's position that correspondence gives the definition of truth, while coherence and pragmatism furnish criteria.

Da Costa might subscribe "to first approximation" (as he likes to say) to this latter view, but he is explicit about the impossibility of *defining* truth, in a strict sense, simply because the definition itself would have to be "true" in the first place! The notion of truth (or falsity) involved in even the most simple descriptive statements is ultimately based on intuition (recall §3.4 above), which is the starting point for any rational elaboration. Furthermore, the author points out the difficulties in characterizing the nature of the correspondence between sentences (or beliefs) and reality.

These difficulties were partially overcome by Tarski's formal definition of truth, where the term "definition" should be taken in a loose sense, as a stipulation of the extension of the concept of truth. Da Costa discusses Tarski's famous "T-schema", an adequacy condition to be satisfied by any satisfactory theory of truth as correspondence: "S" is true in a language $\mathcal{L}$ if and only if S. For example: *"A neve é branca"* is true in the Portuguese language if and only if snow is white. Truth is a metamathematical property of sentences in a given language $\mathcal{L}$ (see Fig. 1). One might mention that Tarski's definition is based entirely on the semantic notion of "satisfaction" of open sentences, and that it proceeds recursively. For simple closed sentences, the T-schema might furnish an adequate definition of truth as correspondence, but Tarski's problem was to generalize this definition to sentences

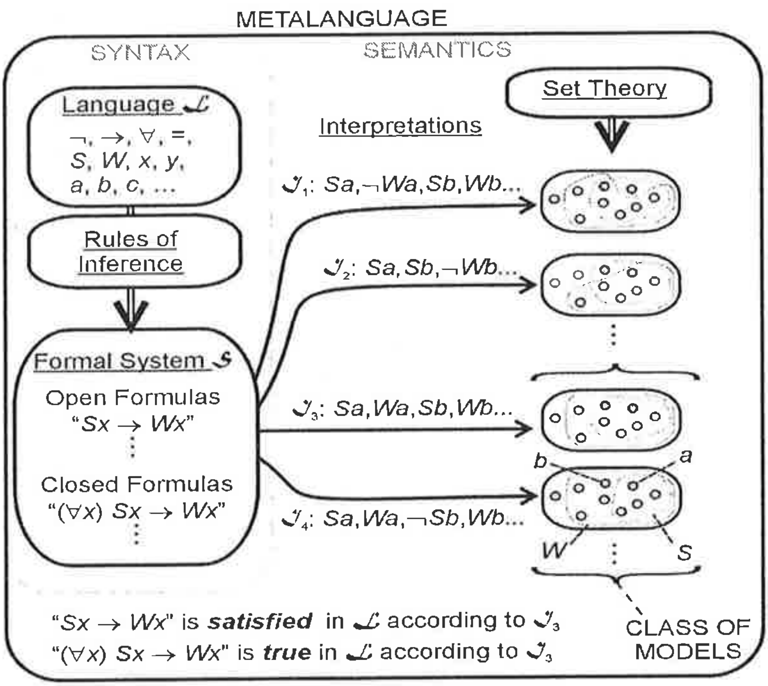

FIGURE 1: A LOGICAL SYSTEM AND ITS INTERPRETATIONS. Given a language $\mathcal{L}$, one may define a formal system $\mathcal{S}$, involving logical and non-logical axioms, and obeying certain rules of inference. Using set theory, one constructs different interpretations of $\mathcal{S}$, such as $\mathcal{I}_3$, which determines that element a *and* b *have properties* S *and* W*, etc. In our example,* Sx *stands for "x is snow",* Wx *for "x is white". According to $\mathcal{I}_3$, object* a *satisfies* "Sx" *and* "$Sx \rightarrow Wx$"*. The sentence* "$(\forall x)\ Sx \rightarrow Wx$" *(translated as "snow is white") is true-in-$\mathcal{L}$ according to $\mathcal{I}_3$. Any interpretation for which all the sentences of $\mathcal{S}$ are true is called a model of $\mathcal{S}$. The truth or falsity of closed formulas in $\mathcal{S}$ are asserted not within the object language $\mathcal{L}$, but in the metalanguage of $\mathcal{L}$. Truth does not depend only on the symbolic rules of a logical system, the syntax, but is a semantical property, depending on the interpretation being considered.*

involving universal and existential quantifiers. Da Costa does not examine Tarski's original definition of truth, but does present a variation of it, discussing the importance of such definitions in logic and also Tarski's undefinability theorem.

The concept of truth provides further examples of foundational pluralism (§4.d): there are numerous alternative theories of truth by correspondence, depending on the system of logic being used (certain multivalued logics don't satisfy the T-schema, for instance) and also on the set theory assumed in the metalanguage[7], while many systems allow self-referential sentences. Da Costa (p. 126) argues once again from this pluralism in the foundations of logic and mathematics to a defense of scientific pluralism (§ 3.5).

6. PRAGMATISM AND PARTIAL TRUTH

Da Costa introduces the theory of partial truth (sections 8-12 of Chapter III) by examining briefly the pragmatic conceptions of truth of C.S. Peirce, William James and H. Vaihinger. Let us take Peirce's famous pragmatic maxim:

> Consider what effects, which might conceivably have practical bearings, we conceive the object of our conception to have. Then, our conception of these effects is the whole of our conception of the object. (Peirce [1878] 1966, p. 124.)

Da Costa and his collaborators, in their first article on the subject, interpreted this conception in the following way:

> Peirce's dictum may obviously be interpreted as implying that the truth (i.e. the pragmatic truth) of an assertion depends on

[7] The scheme in Fig. 1 (and consequently in the other figures) is based on a specific choice of set theory, as da Costa likes to emphasize, and assumes classical semantics.

> the practical effects of it, supposing that they are accepted as true in the ordinary sense of the word "truth".
> [...] a statement – in general a theoretical one – is pragmatically true only when the basic statements it implies are true in the sense of the correspondence conception of truth. (Mikenberg, da Costa & Chuaqui, (1986), p. 202.)

In the book being reviewed, da Costa adds:

> A sentence *S* is pragmatically true, or quasi-true, in a domain of knowledge *D*, if, within certain limits, *S* saves the appearance in *D* or, in *D*, everything takes place as if it were true according to the correspondence theory (da Costa (1997), p. 128-9).

The intuitive idea is clear. Any theoretical scheme – which includes a specification of the domain of application – that generates verified observational statements (taken to be true by correspondence) is defined as pragmatically true. Such a theory may also be true by correspondence, in the sense that all of its observational statements are true by correspondence.

For da Costa, it is meaningless to claim that non-observational theoretical statements are true by correspondence. This places his variety of logical pragmatism[8] quite close to logical positivism, and distant from scientific realism:

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[8] Da Costa seems to prefer the term *critical pragmatism*, in order to distinguish his position from W.V. Quine's logical pragmatism. It is an interesting historical note that Quine spent a semester lecturing in São Paulo in 1942, attracted to the country after writing a review of the first Brazilian book on mathematical logic, written by Vicente Ferreira da Silva. This sabbatical term resulted in a book, *O Sentido da Nova Lógica* (1944), which strongly influenced the young Newton da Costa, together with Russell, Popper, Enriques, and Carnap. Da Costa's first interest, however, was not logic. Fed by his uncle Milton Carneiro (professor of philosophy at the Federal University of Paraná) with Descartes, Durkheim, and Kant, the fundamental question for the 15-year-old Newton (born in 1929) was: "What is knowledge?".

> Indeed, theoretical constructions, for example in the domain of physics, include notions such as probability wave, quark, and phase space, which seem difficult to accept as effectively corresponding to features of reality. They resemble more the categories that we create to subdue experience. (da Costa 1997, p. 114).

As an example of a false theory (in the sense of correspondence) which is pragmatically true, the author mentions the physics of deterministic chaos (plus its domain of application), which is based on classical mechanics (known to be false in a larger domain), but which attracts a great number of researchers and has many succesful applications.

Da Costa's position is that the conception of truth inherent in the empirical sciences is such pragmatic truth. As such, this is not a controversial thesis, and may be accepted by both realists and instrumentalists. What is unacceptable to realists is his denial that theoretical terms may be true by correspondence, but it seems quite feasible to elaborate a realist version of da Costa's logical pragmatism.
Accepting the relevance of pragmatic truth, in the sense presented above, what has now to be investigated is whether the logical description of partial truth presented by da Costa is acceptable or not.

7. THE SEMANTICAL APPROACH TO THEORIES

Instead of repeating the logically precise characterization given in Mikenberg *et al.* (1986), da Costa & French (1990), or da Costa (1997), which the interested reader may easily consult, the reviewer will try to explain the main features of the logical description of partial truth in a sketchy, intuitive fashion, based on Figs. 2 and 3.

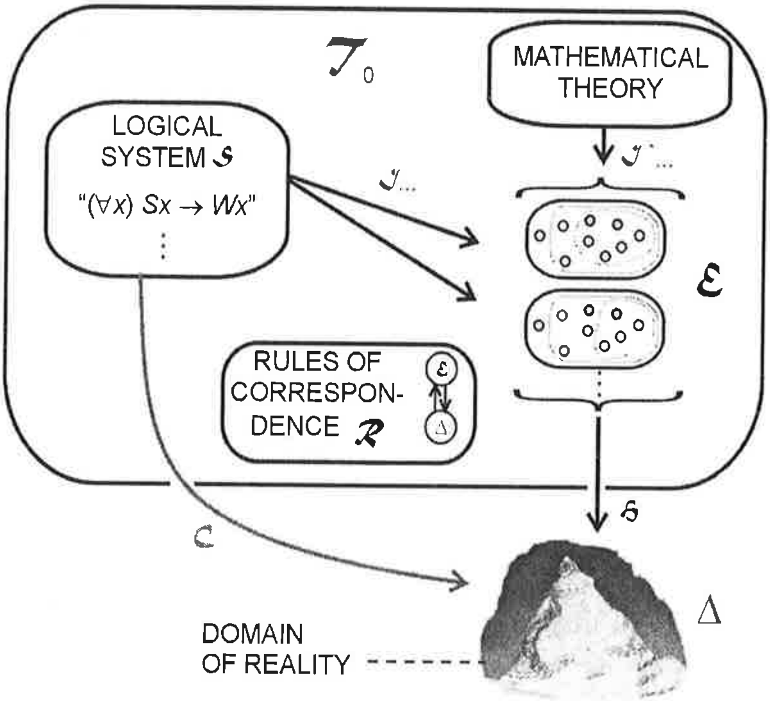

FIGURE 2: THE SEMANTICAL APPROACH TO THE AXIOMATIZATION OF SCIENCE. When considering the logical structure of science, the most natural attitude is to consider that the axioms and theorems of a logical system $\mathcal{S}$ refer directly to reality, by means of a "concrete" interpretation $\mathcal{C}$. This was done by logical positivists in their "syntactical" approach to the axiomatization of science, with the peculiarity that the set of correspondence rules $\mathcal{C}$ was limited to the directly observed reality. The "semantical" approach, on the other hand, emphasizes that $\mathcal{S}$ should be interpreted within set theory by means of an "abstract" interpretation $\mathcal{J}$, as we have seen in Fig. 1. A scientific theory $\mathcal{T}_0$, according to this view, should not be identified with $\mathcal{S}$, but with its class of models, which in physics are also "species of structures" (class of models) $\mathcal{E}$ of a mathematical theory. Some relation of "structural similarity" $\mathcal{H}$ exists between this set of models $\mathcal{E}$ and the domain Δ of the world described by the theory. The set of techniques relating theory and reality are expressed by $\mathcal{R}$. According to da Costa and collaborators, following Suppes, a theory is defined as the set $\langle \mathcal{E}, \Delta, \mathcal{R} \rangle$.

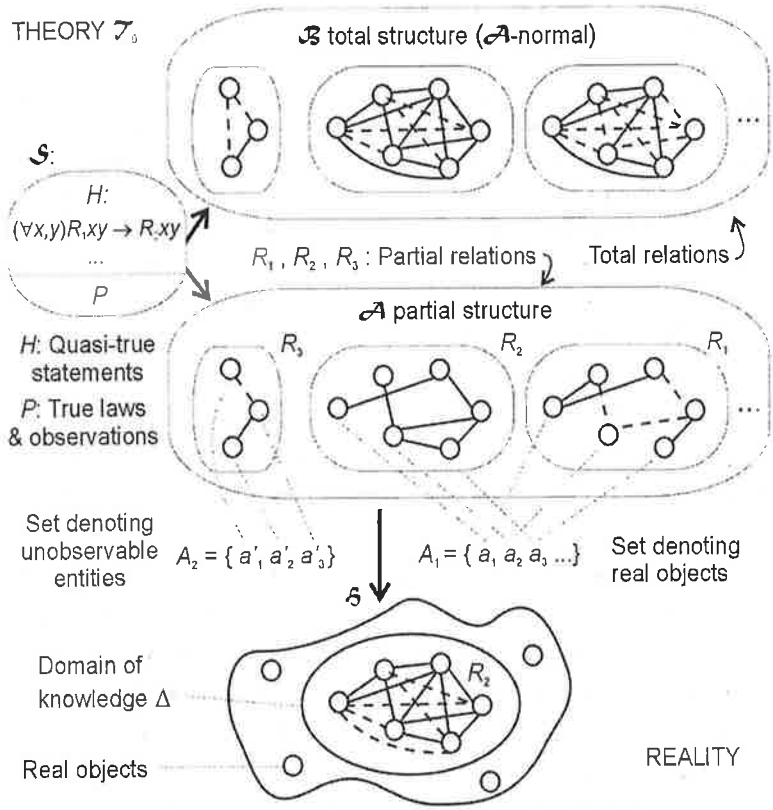

FIGURE 3: THE LOGIC OF PARTIAL TRUTH. According to the semantical view of science of da Costa and collaborators, sentences in $\mathcal{S}$ may be partially true in reference to a domain Δ, although they might be strictly false. In the illustration, each binary symmetric relation R_j between two objects a_i, a_k, is taken to be true *if the objects are connected by a full line (——),* false *if connected by a dashed line (- - -), and* undefined *if unconnected. The formula "$R_1xy \rightarrow R_2xy$" is satisfied in the partial structure $\mathcal{A}$ for every pair of objects for which both relations are defined. The corresponding law "$(\forall x,y)\ R_1xy \rightarrow R_2xy$" is also* partially true, *since there is a total structure $\mathcal{B}$ which extends the partial relations R_i and is consistent with P. If we assume that the instantiations of the total relations R_1 depicted in $\mathcal{B}$ correspond to reality, but that those of R_2 do not, while the instantiations of R_2 illustrated at the level of reality (for which "$R_2a_1a_3$" is false) do, then the above law is strictly false.*

First of all, let us characterize the semantical approach to the axiomatization of scientific theories (Fig. 2), based on da Costa's description in Chapter IV, sections 5-6 (see also da Costa & French (1990), pp. 249-51). What is a scientific theory? Roughly, it is a set of postulates, describing fundamental empirical discoveries, which can be combined with further empirical data so that new predictions can be deduced. It is very natural to represent this by means of a logical system $\mathcal{S}$, of the type we illustrated in Fig. 1. The difference now is that our logical language must refer to reality.

The most simple way of conceiving this relation is interpreting the logical system directly by means of a "concrete interpretation" $\mathcal{C}$, which constitutes the "syntactical approach" to the axiomatization of scientific theories. Observable phenomena such as the color of an atomic emission would correspond to observable terms in the theory, and the relation between observable entities would correspond to empirical laws. Such correspondence rules (which can be identified with $\mathcal{C}$) might be limited to observational terms and laws (logical positivism) or may also involve theoretical terms and laws, which would refer to real but not directly observed entities and processes (scientific realism).

One problem with the syntactical approach is that the same scientific theory might be formulated using different axioms. These different formulations of the same theory, however, conserve the same set of models; this suggests that a scientific theory should be identified with the set of models of a logical system. This "semantical approach" has the additional advantage of being able to incorporate mathematical structures in a natural way, since such structures are usually built from set theory. Fig. 2 illustrates the set of models $\mathcal{E}$ of an appropriate mathematical theory, for instance an infinite dimension complex vector space, which da Costa calls "species of structures", following Bourbaki. In the illustration, such a species of structures is also the class of models of the logical system $\mathcal{S}$. In simple cases, the correspondence $\mathfrak{H}$ between $\mathcal{E}$ and the appropriate domain Δ of reality is an isomorphism, but da Costa argues that in general such a relation is much more complicated (p. 165). Finally, one must include rules of correspondence $\mathcal{R}$ between $\mathcal{E}$ and Δ, which include techniques of preparation and measurement, auxilliary theories, etc.

It is worth noting that da Costa does not reject the syntactical approach, considering these two approaches to the axiomatization of science as different perspectives on the problem, not entirely equivalent but both of interest (pp. 164, 166).

8. THE LOGIC OF PARTIAL TRUTH

Let us turn now to the logic of partial truth (Fig. 3). Consider a theory $\mathcal{T}_0$, which applies to a certain domain of reality Δ. Each object of this domain is represented by elements a_i of a set A_1. (Da Costa takes these objects to be directly observable phenomena, such as spectral lines and tracks in a cloud chamber, and not indirectly observable entities such as hydrogen atoms.) The elements a_i satisfy certain properties, relations and operations, some of which are experimentally well confirmed and some of which are not so firmly established. The well confirmed observations and laws are expressed by a set P of true "primary" propositions.

Another set H consists of propositions that are not directly confirmed, but from which one may deduce true statements from the set P, besides other unconfirmed conclusions. This set H of hypothetical statements is the author's focus of attention. They might be true or they might be false, in the sense of Tarski's correspondence theory of truth, but as far as theory $\mathcal{T}_0$ is concerned, all that matters is that they "save the appearances", i.e. their observational consequences are true. One day, however, their truth-value might be revealed; this might happen when more is known about the domain of reality Δ. This supposes that Δ is not known in a complete way by the present day theory $\mathcal{T}_0$. In set theoretical terms, this "incompleteness" is expressed by certain properties, relations or operations not having a definite value. Restricting ourselves to relations, this amounts to structures known as *partial relations*, which have definite truth-values for certain subsets of elements of A_1, but which are undefined for other subsets. In Fig. 3, for the sake of illustration, we represent binary relations $R_j xy$ which are symmetric, i.e. $R_j\, xy = R_j\, yx$. In the figure, one can see that in the partial structure (or "simple pragmatic structure") $\mathcal{A}$, certain relations are neither true nor false, but are simply undefined, such as $R_1\, a_1\, a_2$.

One basic idea of da Costa and his collaborators is to connect the tentative character of the hypothetical propositions of H to the "incompleteness" of the theory $\mathcal{T}_0$, expressed by undefined relations. Once the truth-values of these relations are known, the truth-value of each $\phi_k \in H$ will be known. But as $\mathcal{T}_0$ stands, the statements in H fulfill an important heuristic role, generating useful consequences: they are *pragmatically* or *partially true*. This, however, depends on the possibility of extending the theory without conflicting with the set of true statements P. Given a certain statement ϕ_k from H, if it is possible to generate a total structure $\mathcal{B}$, simply by attributing a truth-value to all undefined relations in $\mathcal{A}$ (in this case $\mathcal{B}$ is called "$\mathcal{A}$-normal"), in a way that does not contradict ϕ_k and the statements of set P, then ϕ_k is said to be "pragmatically true". Otherwise, it is pragmatically false.

The author does not consider explicitly the situation in which there are more than one pragmatically true sentences. Considering a set of two of them, ϕ_1 and ϕ_2, the total structures associated to ϕ_1 might all be different from those associated to ϕ_2. In

In other words, there would be no total structure associated to the conjunction $\phi_1 \wedge \phi_2$. Still, ϕ_1 and ϕ_2 might lead to correct predictions, so they should be considered pragmatically true. What does da Costa's logical theory of partial truth say about this?

Da Costa completes his formal exposition of pragmatic truth by considering the extension of the language $\mathcal{L}$ of a partial structure $\mathcal{A}$ to a modal language, since the $\mathcal{A}$-normal structures can be interpreted as "possible worlds". Pragmatic truth can then be identified with the modal operator of possibility. This leads to the definition of a "pragmatic theory", which can accommodate contradictory theorems without becoming trivial, placing it in the class of paraconsistent logics (p. 138). This, in turn, reflects the fact that two contradictory propositions may both be pragmatically true within the same scientific theory (we will return to this issue in §12).

In Chapter IV, section 6, da Costa explains how the logical theory of partial truth fits in with the structural (semantical) approach to axiomatization. The $\mathcal{A}$-normal structures are to be identified with the models of $\mathcal{E}$ of the structural approach. The pragmatic definition of scientific theories would therefore involve the triple $\langle \Delta, \mathcal{A}, \mathcal{E} \rangle$, although here da Costa seems to have forgotten the rules of correspondence $\mathcal{R}$, which are so important in establishing a scientific theory, and which is probably the aspect of axiomatization in need of the greatest clarification.

Sections 13-15 of Chapter III discuss some additional issues of the theory of pragmatic truth, and some of the highlights are the following:

(a) *Philosophical neutrality.* Da Costa claims that his theory of partial truth is "philosophically neutral", just as Tarski claimed of his theory of truth. This is probably an exaggeration, but as far as the realist versus positivist debate is concerned, it seems correct to say that both sides may profitably use the theory of da Costa and his collaborators, in spite of the author's anti-realism (recall §6).

(b) *Metalinguistic partial truth.* Da Costa considers uninteresting the fact that the theory of partial truth should itself be considered partially true. From the naturalistic perspective of the reviewer, however, concerned as he is with a "scientific" account of scientific development, such self-reference would be desirable.

(c) *Coherence theory of truth.* Such a characterization of truth, summarized in section 14, amounts to the syntactical version of the theory of partial truth.

(d) *Partial truths in mathematics.* Da Costa divides mathematics into two dimensions. The "external", syntactical dimension, which uses a combinatorial language to construct new results, aspires to truth by correspondence. The "internal", semantical dimension, however, involving models and interpretations, may also make use of pragmatic and coherence conceptions of truths.

9. SCIENTIFIC DEVELOPMENT

Two modes of scientific development are suggested by da Costa, in Chapter III. The first one, mentioned in §8, involves improved empirical knowledge of a certain domain Δ. Notice that the actual development of a scientific field may lead to a total partial structure that is not $\mathcal{A}$-normal, i.e. for which a certain proposition ϕ_k is false. Still, ϕ_k will be considered approximately true within the previous theory $\mathcal{T}_0$, since there exists a possible $\mathcal{A}$-normal extension of it.

The second mode of scientific development involves an *enlargement of the domain of application* of the theory (see also p. 208). If a new theory $\mathcal{T}_1$ is able to describe adequately this enlarged domain, while the original theory $\mathcal{T}_0$ is incapable of doing this, then the latter should be rejected. Still, $\mathcal{T}_0$ will be *approximately true* in the restricted domain Δ (for some further details, see Mikenberg *et al.* (1986), p. 205), and the author will invoke a “principle of correspondence” (p. 159) to describe the fact that, in the limit of Δ (for example, at low velocities), $\mathcal{T}_1$ (relativistic mechanics) approximates $\mathcal{T}_0$ (classical mechanics).

Da Costa (p. 141) introduces the notion of a sentence α_1 being closer to the truth than another α_2. The criteria, however, is syntactical and is close to the proposal made by Popper ((1963), § 10.3). The idea is to compare the number of true consequences in P obtained by adding α_1 to the theory with the number obtained by adding α_2. The one which leads to a greater number of true statements is to be considered closer to the truth. This formulation is attractive, but it has some serious problems. First of all, assuming that α_1 has a greater “truth content relative to P” than α_2 (i.e. implies a greater number of true statements in P), it might happen that α_1 also implies a greater number of false statements than α_2. Wouldn't this “falsity content” also be relevant for determining the approximation to truth? Popper's point of view was that both types of content should be taken into account, so that the degree of approximation to the truth could be measured by the difference between truth content and falsity content. By 1973, however, it became clear that Popper's criterion did not work (see Niiniluoto (1987), pp. 183-92), opening the field for different proposed measures of “truthlikeness”. Da Costa's proposal should take these developments into account.

In Chapter IV (sections 1-4), da Costa returns to issues related to scientific development, claiming that the aim of the empirical sciences is to attain partial truth. Recalling his scientific pluralism (which will be further explored in §12), any sort of conceptual system whatsoever may be constructed and used in science. The empirical consequences of such theories should then be experimentally tested, according to the hypothetico-deductive method. If this comparison is successful, the partial truth of the theory is confirmed. Criteria for confirmation or disconfirmation of hypotheses is furnished by inductive logic (see §11). Once a hypothesis is accepted, after having been shown to be capable of saving the appearences, then it will be forever partially true! "A good theory in Δ, adequately corroborated and resistent to quasi-falsification, is, was, and will eternally be partially true in Δ" (p. 161).

This is the core of da Costa's *cumulative view of scientific development*, which is opposed to the conceptions of Kuhn and also of Popper. Concerning Kuhn, da Costa accepts the existence of paradigm changes and scientific revolutions, but denies that theories within different paradigms are incommensurable (pp. 43, 210).

Da Costa is also explicitely opposed to the Popperian idea that a theory may be falsified (and to the Carnapian notion of confirmation). "There is no falsification, simply because a good theory is not properly falsified, but has, when necessary, its domain of application restricted" (p. 161; see also p. 199). What da Costa seems to be opposed to in this passage is not the methodology of attempting to falsify theories, but the philosophical import given to this procedure. He must admit that a theory that has been falsified *is false*, in the sense of the correspondence theory of truth. His point is that falsification amounts to a restriction of the theory's domain of application, so that the falsified theory remains being pragmatically true and useful. In another passage, da Costa criticizes Popper's methodology at a "psychological" level, because it does not express the actual concerns of scientists, "who don't pursue theories to prove they are false, but to attempt to prove they are true" (p. 51; see also Béziau's preface, p. 12). Such a criticism wouldn't worry Popper, since his methodology is a normative one (describing what scientists should do, not what they actually do), but it does reveal a difference in *emphasis*. The reviewer would say that da Costa ends up agreeing with the norms of the Popperian methodology (see p. 170), especially in light of the critical attitude it brings forth (recall the dimensions of rationality in §3.3, to which we will return), but his emphasis is on the "positive" attainment of partial truth (which is closer to the attitude of the naïve scientist) and not on the "negative" falsification of theories or restriction of domains.

In §3 we mentioned the four dimensions that characterize the rationality of science, according to Chapter I of da Costa's book: concepts, deduction, induction and criticism. In Chapter V he returns to this topic in more detail, but considers only the latter three dimensions. He concludes this interesting discussion (which we will survey in § 12) by proposing an *approximate criterion of demarcation* between science and non-science:

> (*C*) An investigation is scientific if it strives for partial truth rationally, that is, deductively, inductively, and critically (da Costa (1997), p. 204).

The author himself recognizes that such a criterion is vague, failing in certain borderline cases. One interesting aspect of it is that it explicitly depends on the historical moment, since what counts as deduction, induction and methodology depends on the stage of scientific development.

Returning to the problem of how science develops, da Costa considers the problem of the choice between competing theories. It is rational to pick the theory that is best confirmed, or best corroborated, and for da Costa this should be translated into measures of pragmatic probabilities, an essentially Bayesian approach. But what happens when two competing theories explain the same amount of data relative to the same domain of knowledge? The analysis of partial truth and of pragmatic probabilities is not sufficient for establishing which one is best. Other criteria should come into play, according to the author, such as explanatory power, heuristic power, and simplicity (p. 207). Now what do these criteria mean, within da Costa's structural approach? These are considered *pragmatic criteria* by the author (p. 208), but does this mean that they cannot be formalized within the semantical approach? Surely not. The formalization of these additional criteria for theory choice remains as an open problem for the research program that da Costa and his collaborators have inaugurated.

One last methodological issue emphasized by the author, in Chapter V, section 6, concerns Duhem's realization that what is experimentally tested are not single hypotheses, but groups of hypotheses. According to the theory of partial truth, the falsification of a prediction might force the elimination of one among many hypotheses, or might amount to a restriction of the domain of application of a partial structure $\mathfrak{E}$. It is interesting that da Costa does not accept Quine's version of the Duhem thesis, because of its implication that the system of science is one integrated whole (p. 214). As we have seen, and we will return to this point in the following sections, for da Costa science is composed of families of more or less independent cognitive systems.

10. THEORY OF KNOWLEDGE AND THE PRINCIPLES OF SCIENCE

Sections 2-8 and 17 (and notes I-III) of Chapter I survey the standard definition of knowledge as "justified true belief": someone *knows* a sentence p if and only if he *believes* in p, p is *true*, and the belief in p is *justified*. The author presents a simple logical system which is intended to capture the formal dimension of such a characterization of knowledge. This is done by defining logical operators of belief, justification, and truth (the latter in the sense of the theory of correspondence). The resulting system suffers from the usual paradoxes discussed in the 1960's by Gettier, Chisholm, *et al.*, but it appears to be a good starting point for further improvement. Da Costa does not use this epistemic logic anywhere else in the book, although he does refer to it occasionally.

Sections 10-14 consist of a discussion of the classification of the areas of knowledge, especially the formal sciences and the pure sciences. The next topic (sections 18-26) is a personal overview of five traditional problems of the theory of knowledge, as formulated in Johannes Hessen's *Erkenntnistheorie* (1926), widely used in undergraduate classes at the Faculty of Philosophy, Letters, and Human Sciences, University of São Paulo, where da Costa was hired in 1982, after leaving the Institute of Mathematics and Statistics. Some of his views are the following:

(i) Scientific knowledge "is possible", which amounts to a rejection of skepticism and also of relativism.

(ii) Concerning the "origin" of knowledge, the author takes a middle ground between empiricism and rationalism, emphasizing that the features of reality captured by science are essentially mathematical.

(iii) The problem of the "nature" of knowledge involves the debate between realism and idealism, and da Costa seems to favor a form of structural realism. His conception of scientific realism, however, is not at all sophisticated, since he is first of all a logician (and a logician of science), and does not follow the contemporary literature in the philosophy of science (although some of his close collaborators do).

(iv) Knowledge is basically rational, but there is a fundamental place for *intuition* (§3.4), although not of the type that concerned German idealism, an "immediate, non-sensorial, material intuition".

(v) The problem of the "criterion" of truth: in the formal sciences, it is evidence, while in the natural sciences it involves experimental methodology and inductive logic.

The last part of Chapter I concentrates on the logical underpinnings of scientific theories. Da Costa proposes five general principles or norms which define the aims of the natural sciences, four of them based on the traditional problems of the theory of knowledge, summarized in the preceding paragraphs. Besides the principles of (i) possibility, (ii) origin, (iii) nature, and (v) criterion, da Costa presents his "principle of categories", expressing his pluralist approach to knowledge: science is constructed and develops by means of diverse and evolving systems of categories. Although one must agree with the author that such principles (perhaps with modifications) are necessary to science, it is not clear that they are sufficient to characterize science.

In Chapter V, section 5, da Costa presents three additional *metaphysical postulates* assumed in science, which are in fact postulates of realism. The first is named the *Postulate of the External World.* The scientist postulates the existence of a world external to himself and independent of him (ontological realism), to a large extent. Knowledge of this world is the aim of science. The second is the *Postulate of Partial Truth.* By means of the method of science, partial truth, and indirectly, truth, can be attained (epistemological realism). The third, called the *Postulate of Sufficient Reason,* is not very well explained, but seems to be saying that theoretical explanations have a counterpart in reality, corresponding to efficient causes.

11. PRAGMATIC PROBABILITY

Sections 7-12 of Chapter IV approach the problem of the acceptance of a scientific theory by means of the development of a system of inductive logic applicable to pragmatic truth.

By *inductive inference,* da Costa means any form of useful inference that is not deductive, that is not strictly valid in a logical system. After explaining that the rules of inductive inference are non-monotonic, since the acquisition of new knowledge may render incorrect a previously correct induction, da Costa summarizes six classical forms of induction: (i) induction by simple enumeration; (ii) analogy; (iii) statistical inference; (iv) Bacon-Mill methods of elimination; (v) the hypothetico-deductive method; and (vi) probabilistic inference. In the Appendix on inductivism (pp. 189-92), Otávio Bueno characterizes this conception as an "extended inductivism", since da Costa does not restrict the term to the passage from individual statements to universal ones (induction by simple enumeration), as is done by Popper. To understand the inclusion of the hypothetico-deductive method within this extended inductivism, consider that the acceptance of a hypothesis by this method may be later revised when new knowledge is acquired, contrary to what happens in strict deduction within a logical system.

What is the degree of rational belief one should ascribe to the truth of a sentence or theory? The measure of this rational belief is the *probability* of the sentence, which can either be qualitative ("very probable", "probable", "improbable", "very improbable"), comparative ("less probable than", "equally probable as") or metric. Da Costa presents an axiomatization for each of these types of probability, emphasizing of course the latter one, based on Keynes' original axiomatization (1922), for which he derives several theorems.

Now what would be the probability of a scientific theory such as general relativity? If such probability measures the degree of rational belief in the *truth by correspondence* of the theory, then it is practically zero! And this would be the case for all scientific theories we know (unless we have reasons to believe that one of them has attained the "final truth"). Da Costa's solution is to apply probability measures to the degree of rational belief in the *partial truth* of a sentence or theory. Formally, this is done simply by adding the modal operator of possibility "$\Diamond$" to the sentences to which a probability is assigned. We would therefore assign a high probability to the statement that "general relativity is pragmatically true", or to: "it is possible that general relativity is true by correspondence".

With this, da Costa replaces the definition of knowledge as "justified true belief" by "belief that is pragmatically true and justified with high pragmatic probability". The aim of the empirical sciences would be "to attain partial truth with pragmatic probability 1" (p. 184).

12. INCOMPATIBILITIES IN THE EMPIRICAL SCIENCES

Chapter IV considers the empirical sciences, whose aim is, according to the author, to attain partial truth or, when feasible, strict truth. Sections 1-4 (part of which has been examined above) argue that, in practice, scientists usually make use of different and often incompatible theories, which is characteristic of *scientific pluralism* (§3.5). The reviewer would endorse the following quotation presented by the author:

> In fact, to leap from one theory to another is an important part of the art of the physicist. [...] Physical theories, on the contrary, do not need to be logically coherent; their unity is given by the fact that they describe one and the same reality. (Ruelle, (1991), ch. 2.)

Da Costa's point is twofold:

(1) Two mutually incompatible theories (at least one of which cannot be strictly true) are quite often used to describe a physical domain;

(2) A single theory which makes use of inconsistent statements is also occasionally used (such a theory cannot be strictly true, but only pragmatically).

The grand example of this second type is Bohr's theory of the one-electron atom. Da Costa claims that this theory uses incompatible statements because it makes use of Newtonian mechanics and Maxwellian electromagnetism, just like what happens in plasma theory. The reviewer finds it strange to claim that these theories are incompatible. It is true that the equations of Newtonian mechanics are invariant under Galilean transformations, while those of electromagnetism are invariant under Lorentz transformations, which only converge for transformations to reference frames at low velocities. But it is possible to construct a model of the microscopic world which satisfies Newtonian mechanics and which leads to Maxwell's laws, namely Lorentz's 1892 theory of the electron with contraction of lengths which depend on the velocity relative to the aether. The problem of the consistency of classical mechanics and electromagnetism is not simple and should be examined in greater detail. Anyway, as da Costa also points out, Bohr's atomic theory clearly uses incompatible statements because it employs, one the one hand, classical physics with continuously varying quantities, and on the other, the quantum postulate with discontinuous quantities.

A third position seems to be defended by da Costa:

(3) Bohr's principle of complementarity generalized to epistemology, which claims that in certain domains of knowledge one of two inconsistent theories must be used, either one or the other, but never both at the same time.

In order to formalize the use of inconsistent statements within a single theory (type 2), da Costa and his collaborators propose that the underlying logic of such theories be multideductive logic, which is a subclass of paraconsistent logics. Considering the use of different theories for a single domain (type 1), the author also refers to such non-classical logics, concluding that "the logic of physics, as a whole, is in fact paraconsistent" (p. 162). The reviewer tends to agree with the author on this issue (type 1), remembering however that the quotation is referring to the state of science, and not to the state of the natural world.

In Chapter V, section 2, he returns to this topic, arguing that all the conceptual systems composing science cannot be unified by classical logic, i.e. there is no "theoretical monism" based on classical logic. This is stated more as a practical limitation, and should not be understood as a claim that reality is not in principle amenable to classical logic and to the correspondence theory of truth.

At this point da Costa introduces a fourth thesis in favor of scientific pluralism:

(4) Certain domains of reality cannot be described by classical logic, the best known example being quantum mechanics, whose underlying logic is non-distributive. "We believe that maybe it is not possible to give quantum mechanics a foundation without a change of logic" (p. 202).

The reviewer disagrees with this last claim. The conclusion that the logic underlying quantum physics is non-distributive depends on the adoption of certain interpretative assumptions. Other interpretations might be chosen that do not lead to this conclusion (although every interpretation has its own "conceptual anomalies"), as I will illustrate in the following section.

13. LOGIC AND PHYSICS

In Chapter V, da Costa presents two significant results in the foundations of physics, the first of which exemplifies the usefulness of axiomatization for physics. We will also make a brief comment on quasi-sets, and then consider in greater detail a fourth example of the connection between logic and physics, which involves quantum logic.

The first result was obtained in 1991 by da Costa himself, in collaboration with Francisco Dória. This result is arguably da Costa's most important result since his pioneering work in paraconsistent logic. Using an axiomatization of classical mechanics, they proved that there is no algorithmic method (applicable to every system) to decide whether a system of diferential equations is chaotic or not. This constitutes an extension of Gödel's incompleteness theorem to physical theories, where the undecidable propositions have physical content.

The second result described in Chapter V involves the Sitnikov-Alekseev theorem in celestial mechanics. This theorem, discussed by Moser (1973), describes the possible trajectories of a very simple three-body system in classical mechanics. Each trajectory may be described by an ordered set of integers, each successive integer measuring the time it takes for a test body of negligible mass to pass successively through the center of mass of the two larger bodies. The surprising result is that above a certain order of the succession of integers, *any* sequence of integers describes a possible trajectory.

This is the content of the Sitnikov-Alekseev theorem. Da Costa goes a step further. Since *any* sequence describes a possible trajectory (defined by the initial conditions of the test particle), then also a *random* sequence describes a trajectory. Now the theory of algorithmic complexity (developed independently by Solomonoff, Kolmogorov, and Chaitin) defines a random sequence as that for which there is no algorithm for generating it which is smaller in size (defined in informational terms, see Chaitin (1975)) than the sequence itself. So da Costa concludes that there are certain initial conditions for which:

> Prediction is mathematically impossible: a typical deterministic system (classical mechanics is deterministic) generates unpredictable phenomena. Thus, determinism does not imply predictability.
> [...] Chaos theory in dynamical systems shows that prediction is sometimes impossible in practice. The result we have just discussed, on the other hand, shows that the impossibility of prediction also takes place for reasons of theoretical order. (Da Costa (1997), p. 221.)

The reviewer does not agree with the author's conclusion. Given the equation of motion describing the system and the initial conditions, any trajectory can in principle be calculated, even those corresponding to random sequences (in the sense of algorithmic complexity theory). In this case, what happens is that the algorithmic complexity for calculating the sequence is greater than the complexity of the sequence itself, but there is nothing in principle preventing the scientist from performing the laborious calculations. One should not confuse the algorithmic definition of randomness with the criterion used in physics, which is associated to the physical process of generation of the sequence.

A third application of logic to physics is the idea of quasi-sets, developed by da Costa's former student Décio Krause. The physical motivation for this theory is the description of elementary particles in quantum mechanics. Even when two particles having the same properties (mass, charge, spin, etc.) are indistinguishable (sharing for example the same position at the same time), they nevertheless are not "identical", since we still have *two* particles. Thus, Leibniz's principle of the identity of indiscernibles is allegedly violated. Krause's idea was to devise a version of set theory in which this violation also takes place, and then use it as a foundation for quantum mechanics. Such a theory has been called "quasi-set theory", and the associated logic is non-reflexive (recall §4.*iv*).

The problem of indistinguishable particles is a complex of problems involving many concepts starting with the letter "i": intrinsic identity, identity of properties, indistinguishability, individuality, interference terms, etc. I have yet to see a definitive philosophical review of such complex of problems in the literature, and can safely say that I am utterly confused with regard to these problems. The motivations and quotations presented by da Costa (pp. 88-9, 120) and by Krause, in his elegant appendix (p. 62-5), also convey a certain amount of confusion on this issue. This, however, does not remove the great interest that their work on quasi-sets has for the foundations of quantum mechanics.

The fourth topic to be examined involves the notion of "underlying logic" which we have previously encountered (§3.6, 10.*iv*). In fact, in the preface to *O Conhecimento Científico*, Jean-Yves Béziau refers to three "pragmatic principles of reason" which da Costa presented in a previous book (da Costa (1980)). (1) *Systematization Principle*: Reason is always expressed by means of a logic. (2) *Unicity Principle*: In a given context, the underlying logic is unique. (3) *Adequacy Principle*: The logic underlying a given context must be the one that best adapts to it.

These are the principles justifying the claim that the domain of quantum physics is ruled by non-classical logic.

> [We are led to believe that] in the domain [of the mechanics of quanta] the standard logical norms are defeated. [...]
> [...] we think that maybe it is not possible to lay the foundations of quantum mechanics without a change of logic. (Da Costa (1997), pp. 201-2.)

This is a widespread claim, especially among logicians, but it should be pointed out that such a conclusion derives from a specific interpretation of quantum mechanics. It is quite easy to interpret quantum theory without abandoning classical logic. One way is to conceive the wave function as a wave in a higher-dimension configuration space. One may then either suppose that such a wave is subject to non-local collapses that accompany measurements (a wave interpretation) or that there is a particle associated with such a non-local "pilot-wave" (the de Broglie-Bohm dualist view). These views indeed lead to strange pictures of the world, but nonetheless this is done within the framework of classical logic.

Da Costa presents a short argument in favor of the view that the underlying logic of the microscopic realm is non-distributive (similar for instance to the one in Hughes, (1981)).

> The spin of electrons [or, rather, neutral atoms] is always measured along a direction. On the other hand, in virtue of Heisenberg's indeterminacy principle, one cannot simultaneously measure the spin in two different directions. [...]
> [1] [Let us suppose] that the spin is polarized along the x axis, possessing value +½ (which is easy to obtain experimentally). Thus, the proposition (α): "The beam has spin +½ in the x direction" is true.
> [2] On the other hand, the propositions (β) "The beam has spin +½ in the y direction" and (γ) "The beam has spin –½ in the y direction", where $x \neq y$, are such that $\beta \vee \gamma$ evidently is true.
> [3] Therefore, so is $\alpha \wedge (\beta \vee \gamma)$.
> [4] Applying the distributive law $\alpha \wedge (\beta \vee \gamma) \leftrightarrow ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$, one finds that $(\alpha \wedge \beta) \vee (\alpha \wedge \gamma)$ is also valid.
> [5] However, since $x \neq y$, it follows by Heisenberg's principle that this last proposition must be false or not make sense.
> [6] We are led to the conclusion that the distributive law is not valid in the quantum world, since the other assumptions used are apparently harmless. (Da Costa (1997), pp. 201-2.)

What is not explicit in this argument are the interpretative assumptions that are adopted. In step [2], everyone agrees that a measurement of the spin component will yield either the values +½ or –½. But to infer from this that the *value* possessed by the particle right before the measurement is either +½ or –½ (as is done in propositions β and γ) involves an additional hypothesis, which is usually called the "assumption of faithful measurements". A wave interpretation rejects this assumption, because it conceives the quantum object as an extended wave, which in general does not possess well-defined values for position or spin before a measurement. According to the wave interpretation, proposition β is false and γ is false, so it is not the case that "$\beta \vee \gamma$ evidently is true".

On the other hand, the class of interpretations that might be called "corpuscular" tends to accept the assumption of faithful measurements. This is the case for instance of the "ensemble interpretation" proposed by Ballentine (for a discussion of all these assumptions and interpretations, see Pessoa (1998)). Curiously enough, this view also believes that a single particle possesses simultaneously well-defined values for the spin components in both x and y directions, so that the proposition $(\alpha \wedge \beta)$ might be true. The uncertainty principle would be a statistical limitation, reflecting the impossibility of preparing identical microscopic states, and not an ontological limitation on possessed values. So a proponent of the ensemble interpretation would reject step [5], while the wave interpretation would accept it.

The upshot is that the use of quantum logic to describe the world is consistent but is different from the other interpretations of quantum theory, in that it accepts the assumption of faithful measurements (step [2]) and the ontological version of the uncertainty principle (step [5]), while rejecting the distributive law (step [4]).

This conclusion is a far cry from the claim that "it is not possible to lay the foundations of quantum mechanics without a change of logic". There is no reason for us to conclude that the unique underlying logic of the microscopic world is non-distributive.

14. CONCLUSION

The central theme of the book is da Costa's theory of partial or pragmatic truth. The more qualitative descriptions of pragmatic truth are quite reasonable, the noteworthy feature being the coexistence of a pragmatic theory of truth with a correspondence conception of truth for observational statements. Another well-made point is the author's arguments in favor of the plurality of science.

What has not yet convinced the reviewer is the adequacy, for actual science, of the logical description of pragmatic truth in terms of partial structures. The logical description is rigorous, elegant and interesting, but such abstract theoretical description of science is very hard to test *vis-à-vis* the history of science. Much work has to be done to compare da Costa's logical theory of science with the actual structure(s) of science.

In the reviewer's opinion, where the author's theory of pragmatic truth is most wanting is in its pragmatic side. Since the book is a description of scientific knowledge, then (contrary to the author's opinion) his metatheory is supposed to "save the appearances" with regard to science, in such a way that his metatheory. should also be subject to the labels "pragmatically true" or "false". The semantical approach to scientific theories and the theory of partial truth should be corrected and perfected while comparison with the actual history of science takes place (for an initial step in such comparison, see French & Ladyman, (1997)).

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