Map of Interpretations of Quantum Theory

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Introduction

This paper is a general study of the interpretations of quantum theory. A classification of these interpretations is proposed, according to an epistemological criterion (positivist or realist) and an ontological one (corpuscular, undulatory, dualist, or without ontology), and an intentional-emotional aspect is also considered. One then considers how four general groups of interpretations answer six questions related to experiments in quantum physics. A survey of the literature concerning these four broad interpretative groups is made, resulting in a "map" involving around forty interpretations of quantum theory.

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1. General Considerations

Quantum theory, i.e., the physics of the microscopic world, has a remarkable aspect associated to it, which is the existence of dozens of different "interpretations". Those who have some familiarity with this theory know that there is an "orthodox" interpretation, and that it is opposed to an interpretation with "hidden variables". The popularization literature makes frequent reference to a "many-worlds" interpretation, and in discussions about non-locality one sometimes writes that the interpretation Einstein gave to the world would be wrong.

How is it possible that there be so many different interpretations for a theory that is considered so fundamental? A bit of reflection shows that this situation, far from being pathological, should be considered typical. An *interpretation* is usually understood as a set of theses or images that are added to the minimal formalism of a theory, without affecting in any way the observational predictions of the theory[1]. These theses make statements about the reality existing behind the observed phenomena, or furnish norms about the inadequacy of making such statements. Clearly, an interpretation is equivalent to a philosophical or metaphysical stance, which the scientist is free to choose.

The fact that quantum theory refers to a domain of reality which is very distant from us (and did not play a selective role in the biological evolution of our cognitive apparatus) makes us consider it counterintuitive; since it is located at the limits of our knowledge, it is difficult to test any conjecture concerning the reality that lies behind our tenuous experimental measurements. Thus, it is natural that there be a great number of hypothetical constructions concerning the nature of this reality which hides behind the observations. In other words, there is a strong subdetermination of interpretation by the minimal formalism of the theory.

Once we have a general theory which is successful in making predictions and in explaining all kinds of measurements, the first guide for postulating what should be the nature of the underlying reality is the structure of the theory itself. If the theory makes use of a mathematical entity which is analogous to a wave, like the wave function $\psi(r,t)$ of Schrödinger's wave mechanics, then the "natural" interpretation of this theory is that there be a referent (in reality) for this wave function. There are other approaches to non-relativistic quantum mechanics that furnish the very same experimental predictions as wave mechanics, like matrix mechanics or Feynman's sum over histories. There are proofs that these approaches are mathematically equivalent, but still each approach "suggests", by means of the mathematical entities that are emphasized (waves, trajectories, possible trajectories), what are the real entities that have priority. Each mathematical formalism *suggests* a different ontology, each one has a different natural interpretation.

Still, there is nothing that forces a physicist that works with wave functions to believe or defend that such waves exist in reality. The "official" interpretation adopted by a scientist does not have to reflect the natural interpretation suggested by the theory[2]. In effect, there is nothing to force a physicist to defend any thesis whatsoever (concerning non-observable reality). If, in fact, he adopts this position of suspending his judgement about reality, that does not mean, however, that he lacks an interpretation concerning the theory, but that he adopts an interpretation that rejects associating a picture of the world to any non-observable part of reality. This attitude is known as *positivism* or, more precisely, as "descriptivism" (according to this view, science should only attempt to describe the observed reality, being "meaningless" to talk about that which is not observable). The orthodox interpretations of quantum theory are to a large extent positivist, while most of the alternative interpretations assert something about non-observable reality, an attitude that is known as *realism*. Any

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[1] It may happen that an interpretation makes predictions that are in disagreement with a theory, so that in this case one should speak of a "different theory"; however, if the disagreement is so small that one cannot make a crucial experiment to chose between the theories, then it is customary to consider that the different theory is also an "interpretation".

[2] On the other hand, one may argue that there are "private interpretations" that the scientist uses, even without noticing, during his or her work, and that may differ from the "official interpretation" which he or she adopts publically (see Montenegro & Pessoa, 2002).

interpretation may be analyzed from the perpective of its degree of positivism/realism. In this paper, a classification of the interpretations of quantum theory is proposed, based on this distinction.

A second criterion for classifying interpretations concerns the proposed *ontology*. In the case of quantum theory, a fundamental ontological distinction is between particle and wave interpretations. This distinction reflects the more general dichotomy between "well-defined propoerties" and "smeared or fuzzy properties". What we call "wave" or "undulatory" interpretations (following Reichenbach, 1944) should be understood as views that do not attribute well-defined properties to certain quantum-mechanical magnitudes, such as position. What we call "particle" or "corpuscular" interpretations include views that attribute simultaneously well-defined values to any observable, including energy or spin component.

Most of the interpretations of quantum theory answer in a clear way to the following questions: "are there particles?", "are there waves?". Thus, there are three broad interpretative groups: *particle*, *wave* and *dualism* (views that accept the existence of both), besides some approaches that avoid any ontological commitment. In this paper, a classification of all the interpretations of quantum theory is proposed, based on how each of them is located along the epistemological (positivism or realism) and the ontological axes (particle, wave, dualism or without ontology).

There is, however, a third axis that would be significant for classifying the interpretations, but whose elusive nature makes it difficult to apply. It consists of the "intentional", or even "emotional" aspect, that people attach to their interpretative positions. There are individuals who defend emphatically or even aggressively an interpretation, and the emotionally laden debate involving two or more parties may result in a "scientific controversy", which may even affect the professional or social levels. We will not use the *intentional-emotional* aspect in our classification of interpretations, although its relevance should be noted.

Consider the following interesting example of how the intentional-emotional aspect affects the cognitive. Some authors propose new formalisms for quantum theory, introducing new concepts that might suggest an original "natural interpretation". However, if such authors are not interested in proposing a new interpretation, the theory is usually seen as part of the orthodox interpretation. A typical example is the approach of the Wigner distribution (see, for example, Freyberger & Schleich, 1997), which introduces the concept of "negative probability". The positivist attitude of Wigner was to consider that such a concept is only a mathematical instrument, but if he had had a more realist attitude (concerning the natural interpretation of his approach), maybe he could have defended a "realism of potentialities" in which such a concept would refer to the "degree of impossibility" of a situation (Feynman, 1987). In other words, a more thorough study of interpretations should consider not only situations in which scientists *declare* that thay are presenting an interpretation, but also cases in which they don't but in which they *could have declared.*

2. Four Broad Interpretative Groups

Following the comments made in the last section, concerning the classification of the interpretations based on the epistemological (positivism or realism) and ontological (particle, wave, or dualist) axes, one can form four broad groups of interpretations of quantum theory. In each of them, we will mention a "naïve" version, which have been used in Pessoa (2003) for an initial contact of physics students with the theory.

(1) *Wave Interpretation (realist).* This point of view considers that the quantum mechanical wave function corresponds to a reality, an undulatory or smeared out (fuzzy)

reality, or maybe a potentiality. An undulatory view was explicitely defended by Erwin Schrödinger, but it was extremely hard for him to account for phenomena withou the notion of "collapse". In a naïve version of the wave interpretation, the reality to which the wave function refers would suffer collapses every time it interacted with a measurement apparatus. A conceptual problem is that such collapses are "non-local", that is, involve effects that propagate in an instantaneous way (see Einstein, in Solvay, 1928, p. 254). This view is close to that of John von Neumann, with the difference that the latter did not associate the wave function with reality (he had a positivist attitude: the wave function only represents our knowledge), so that non-locality was not problematic. The relative states interpretation of Everett (1957), the decoherence approach of Zeh (1993), and the spontaneous localization view (Ghirardi et al., 1986) are other examples of realist wave interpretations.

(2) *Particle Interpretation (realist).* This is the view according to which the microscopic entities (or at least those with nonzero rest mass) are particles, without an associated wave. This positition was explicitely defended by Alfred Landé (1965-75), within the statistical ensemble interpretation. The great difficulty for the particle interpretation is to explain the interference patterns obtained in experiments with electrons. In spite of this problem not being satisfactorily overcome, it is very common to find particle interpretations in the literature and also, in a more naïve way, between students. Interpretations that attribute simultaneously well-defined values to incompatible observables (like position and mometum), and that don't introduce smeared magnitudes, are also classified as "corpuscular". An example is the interpretation implicit in the use of quantum logic.

(3) *Realist Dualist Interpretation.* This interpretation was originally formulated by Louis de Broglie, in his "pilot wave" theory, and extended by David Bohm (1952) to include also the measurement apparatus. The quantum object divides in two parts: a particle with well-defined trajectory (but generally unknown), and as associated wave (or a "quantum potential"). The probability for the particle to propagate in a certain dirction depends on the amplitude of the associated wave, so that in regions where the waves cancel out there is no particle. In the naíve level of an introductory course, this approach is free of the problem of non-locality, and the only conceptual problem is the existence of "empty waves", that don't carry energy. The problem of non-locality only appears when two correlated particles are considered, as shown by John S. Bell.

(4) *Positivist Dualist Interpretation.* This expression especially denotes the complementarity interpretation of Niels Bohr (1928), which identifies a limitation in our capacity of representing macroscopic reality. According to the experiment being made, one may use either a corpuscular description, or a wave picture, but never both at the same time (these excludent aspects, however, "exaust" the description of the object). This does not mean, however, that the quantum object *is* a corpuscle or *is* a wave. According to any positivist interpretation (in the context of physics), one can only assert the existence of observed entities. To assert, for example, that "an unobserved electron suffers a collapse" would be meaningless. A wave phenomenon is characterized by the measurement of an interference pattern, and a corpuscular one by the possibility of inferring (or "retrodicting") a well-defined trajectory. The point-like aspect of every detection (considered by interpretation 2 as the best evidence for the corpuscular nature of quantum objects), which occurs even in wave phenomena, is considered the fundamental principle of quantum theory, and was called the "quantum postulate" by Bohr. There are many variations of this approach, constituting the so-called "orthodox" interpretations. More recently, one may mention the consistent histories interpretations of R.B. Griffiths (1984) and Omnès (1992).

3. Key Questions for distinguishing the Interpretations

A criterion for distinguishing interpretations is to take note of the answers given by each one for different questions. We have developed this exercise in Pessoa (2003), and in what follows we represent some of the questions examined (in Pessoa, 1998, we have already examined these same questions, explaining with some detail experiments with electrons).

3.1 Two Slit Experiment

How to explain the behavior of a quantum, like a photon or an electron, in the two slit experiment? On the one hand, the photon or electron behaves like a particle, as it is detected in a point-like manner; on the other, it behaves like a wave, since the probability of it falling on each point follows an interference pattern. But how is it possible that an entity be, at the same time, wave and particle, if such attributes are contradictory?

Question I: How to explain the two slit experiment for a single quantum?

(1) *Wave Interpretation.* The photon or electron that passes through the two slits is, in reality, a wave, not a particle. In this way, it is easy to explain the formation of the interference pattern on the detecting screen. The appearance of a point on the detection screen occurs because of a "collapse" of the wave, which during the measurement is forced into a very narrow "wave packet", which has the appearance of a point particle.

(2) *Particle Interpretation.* The photon or electron is in reality a particle, which is manifest in the point-like outcome of the detection. There is no associated wave: the interference pattern must be explained as the result of the momentum exchange between electron and diffraction grating (or of any other property involving the device used to separate and recombine the beam).

(3) *Realist Dualist Interpretation.* In reality there exists a particle (with well-defined trajectory) *and* an associated wave (which doesn't carry any energy), as postulated by L. de Broglie (1926) in his "pilot wave" theory. The probability of a particle propagating in a certain direction depends on the amplitude of the associated wave, so that in regions where the waves cancel out, there can't be a particle. This explains in a natural way the appearance of interference fringes.

(4) *Positivist Dualist Interpretation.* According to Niels Bohr's complementarity interpretation, the "phenomenon" in question is undulatory (that is, the conceptual framework we use is in accordance with the physics of waves), and not corpuscular (we cannot infer the past trajectory of a detected quantum). The point-like aspect that we observe as a result of detection is due to the "quantum postulate" mentioned above, which claims the existence of an *essential discontinuity* (an indivisibility) in any atomic process, as for example in the ionization of atoms in the detection screen.

3.2 Mach-Zehnder Interferometer

Instead of using a double slit, it is possible to observe an interference pattern with a Mach-Zehnder interferometer. In this apparatus, developed for the use of light (there is a version for electrons, see Pessoa, 1998), one splits the beam in two by means of a half-silvered mirror S_1, leading to paths A and B. These are then recombined in another half-silvered mirror, S_2. The result, in the case of perfect alignment, is that the whole beam unites again in a certain direction D_1, while in the other available direction, D_2, it disappears completely (destructive interference) (see Pessoa, 2003, ch. 2).

What happens when only *one* photon or electron enters the interferometer? Quantum theory furnishes a simple answer: it will be detected with probability 1 (assuming perfectly efficient detectors and neglecting losses) in D_1 and with probability 0 in D_2. But what happens when the photon or electron is located *inside* the interferometer, before being detected? In this case, each interpretation will give a different answer.

Question II: What happens when the electron is inside the interferometer?

(1) *Wave Interpretation.* The electron, which can be identified with a wave packet propagating in space, splits in two after the first half-silvered mirror S_1, in accordance with what classical physics of waves would predict. These "half electrons" would then recombine in S_2, and due to the destructive interference that occurs in the direction of D_2, the whole packet arrives in D_1. What remains to be explained is why half electrons are never detected (see the following section).

(2) *Particle Interpretation.* Since the electron can never be split, it *either* follows path *A* (and nothing goes along path *B*), *or* path *B* (and nothing goes along *A*). However, if the electron moves with certainty along path *A* (which can be guaranteed by removing S_1), the probability of it being detected in D_2 is different from zero; and if it moves along *B* (introducing a reflector of electrons in S_1), the probability is also different from zero. However, the probability of detection in D_2 is 0! Therefore, one cannot simply say that the electron went *either* by *A or* by *B*. A way out of this dilemma is to argue that the logic at the quantum level is of a non-classical type, thus invalidating the preceding argument (see Pessoa, 2004).

(3) *Realist Dualist Interpretation.* This point of view also asserts that the electron is not split, but it escapes the aforementioned dilemma by postulating that the wave associated to the corpuscle splits in two in S_1 and recombines in S_2, leading to interference. The particle behaves like a "surfer" who can only move where there are waves; since the waves cancl out in the direction of D_2, the electron is forced to surf towards D_1.

(4) *Positivist Dualist Interpretation.* According to the view of Bohr, a phenomenon may be either undulatory or corpuscular, but never both at the same time. The examined experiment is a wave phenomenon, therefore it is *meaningless* to ask where the electron is.

3.3 Anti-Correlation Experiment

The two experiments previously examined are considered "wave phenomena" by the complementarity interpretation. Let us now see how the different interpretations explain a "corpuscular phenomenon".

Consider a beam of light which falls on a *single* half-silvered mirror S_1. Naturally, the beam will split in equal portions among paths *A* or *B*. It so happens that if there is only one photon, it will be detected *either* in D_A *or* em D_B (assuming perfectly efficient detectors), but never in both at the same time. This phenomenon is known as "anti-correlation". In other words, when detected, the photon maintains its individuality and does not have its energy divided. How do the different interpretations explain this phenomenon?

Question III: How to explain the anti-correlation experiment?

(1) *Wave Interpretation.* After reaching S_1, the wave packet associated to the photon splits in two, which is expressed by the wave function $\psi_A + \psi_B$. However, once the photon is detected, say in D_A, then the probability of detection in D_B becomes zero instantaneously! The initial state is reduced, in this case, to ψ_A. Since, in this interpretation, the state corresponds to a "real" probability wave, one concludes that a process of *collapse* of the wave packet took place.

(2) *Particle Interpretation*. In this case the explanation is straightforward: the particle simply followed one of the possible trajectories (*A* or *B*), ending up in one of the detectors, D_A ou D_B . One does not have to speak of "collapse".

(3) *Realist Dualist Interpretation*. This view also considers that, after S_1, the particle follows one of the trajectories *A* or *B*, falling on the correpsonding detector. But there would exist an associated wave, which splits in two. The part that is not detected would be an "empty wave" that does not carry energy and cannot be detected. This leads to a proliferation of entities, but without any undesirable observational consequence.

(4) *Positivist Dualist Interpretation*. Once the measurement is completed, the complementarity interpretation would consider this phenomenon as being corpuscular. Therefore, the photon can be considered a particle that followed a well-defined trajectory. Such an inference concerning the past history of the detected quantum is called *retrodiction*. When examining the uncertainty principle, both Bohr ([1928] 1934, p. 66) and Heisenberg (1930, pp. 20, 25) emphasized that retrodiction is a metaphysical hypothesis that needs not be accepted (in spite of its acceptance not leading to any contradictions); however, when he defined "phenomenon", Bohr ended up making implicit use of this hypothesis.

3.4 The Quantum-Mechanical State

A central concept to be interpreted is that of a "state" $|\psi\rangle$. To what does this theoretical term refer? Let us see how each point of view approaches this issue.

Question IV: To what does the quantum state refer?

(1) *Wave Interpretation*. Interprets $|\psi\rangle$ in a "literal" way, attributing reality to the state or to the wave function, and claiming that nothing else exists, besides what is described by the quantum-mechanical formalism. But what kind of reality is this? It is not an "actualized" reality, that we can observe directly. It is an intermediate reality, a *potentiality*, that establishes only probabilities, but that notwithstanding evolves in time as a wave. The biggest problem of this interpretation of state is that, for *N* quantum objects, the wave function is defined in a 3*N*-dimensional configuration space: what would that mean, a 3*N*-dimensional reality?

(2) *Particle Interpretation*. The state $|\psi\rangle$ is an essentially statistical description, that represents an average over all possible positions of the particle. In technical language, the state represents a statistical "ensemble", associated to an experimental preparation procedure. Thus, this view considers that the quantum state represents an *incomplete* description of an individual object.

(3) *Realist Dualist Interpretation*. Considers that there exist "hidden variables" behind the description in terms of states; such variables are the positions and velocities of the particles. The state $|\psi\rangle$ expresses a real field in 3 dimensions that "guides" the particles. Such "pilot wave", however, does not carry energy, which is concentrated in the particle. The description given by the quantum state would be incomplete, and would only be completed with the introduction of the hidden parameters.

(4) *Positivist Dualist Interpretation*. Considers that the state $|\psi\rangle$ is only a mathematical instrument for making calculations and obtaining predictions (this view is called "instrumentalism"). Heisenberg (1958, p. 55) expressed this in a radical way by writing that the discontinuous change in the wave function is a "discontinuous change in our knowledge", which amounts to an *epistemic* view of the quantum state. The statistical ensemble interpretation (item 2 above) also shares this view; the difference, however, is that the complementarity interpretation considers that the quantum state is the most "complete" description of an individual quantum object. Emphasis is also given to *relationism*: the reality of a quantum phenomenon only exists in the relation between microscopic object and

measurement apparatus.

3.5 Measurements in Quantum Physics

The historian of science Max Jammer defends the thesis that Bohr, before adopting the relationist stance, had "interactionalist" conception: in general, a particle only acquires a well-defined value p_x of momentum (for exemple) after interacting with the measurement apparatus and the outcome p_x being obtained. Pascual Jordan (1934) expressed this in a more radical way: "we ourselves produce the results of the experiment" (see Jammer, 1974, p. 161).

There is a certain consensus that the magnitude that is directly measured, either in measurements in classical or quantum physics, is *position* (velocity, momentum, etc. would be indirectly measured, from direct measurements of position and from the counting of events). Let us see, in this section, how the different interpretations consider the measurement of a magnitude such as the position x.

Question V: What can be said about the prior existence of a measured value of position x?

(1) *Wave Interpretation.* In the case in which the quantum object is in a superposition of position eigenstates (that is, the wave function $\psi(x)$ is not sharply peaked around a value of x), then one cannot attribute a well-defined value for position. After the measurement, assuming that the value x_0 was obtained, a collapse of the spread out wave to one sharply peaked around x_0 occurs (according to the projection postulate). After the measurement, therefore, one may attribute a well-defined value for position, but not before.

(2) *Particle Interpretation.* In this interpretation, it is common to accept that the position measurements are *faithful*: they reveal the value of the position possessed by the particle before the measurement process. Furthermore, immediately after the measurement the position of the particle remains the same. However, in order to adequately explain experiments in which incompatible observables are measured in succession, one must admit that the measurement of position *disturbs* in an uncontrollable and unpredictable way the momentum of the particle. This, in fact, was the interpretation adopted by Heisenberg in his semi-classical derivation of the uncertainty principle (see following section).

(3) *Realist Dualist Interpretation.* According to this view, position measurements are faithful, revealing the value possessed before the measurement. Such a measurement introduces an instantaneous change in the associated wave, which affects the momentum in an unpredictable way (the change in the wave would depend on the microscopic state of the measurement apparatus, which is never known by the scientist).

(4) *Positivist Dualist Interpretation.* For an interpretation that tends to attribute reality only to what is observed, strictly speaking it is meaningless to ask what the position of the particle was before measurement. This is expressed by the "interactionalism" mentioned above with the quotation from Jordan. However, in its "relationist" version, the complementarity interpretation ends up adopting retrodiction. In this case, therefore, it is plausible to say, *after* the detection of a quantum in a certain position x_0 (for either particle or wave phenomena), that the position of the quantum object right before the measurement was x_0 (but *before* the measurement it is incorrect to say that "it has a well-defined but unknown position", since the detector may be quickly removed and an interference between the different paths may be introduced).

3.6 Interpretations of the Uncertainty Principle

To conclude this chapter, let us examine how the different interpretative groups account for the *uncertainty relations* for pairs of "incompatible" magnitudes, originally derived in 1927 by Heisenberg. To simplify the discussion, we will consider the relation involving position x and the momentum component p_x: $\Delta x \cdot \Delta p_x \geq \hbar/2$.

Question VI: What is the meaning of the uncertainty relation?

(1) *Wave Interpretation*. Attributing reality only to the wave packet (without postulating the existence of point particles), Δx measures the extension of the packet, indicating that the position x of the quantum object is undetermined or not well-defined by a quantity Δx. Thus, the relation expresses a principle of *indetermination*: if x is well-defined, p_x is not well-defined, and vice-versa.

(2) *Particle Interpretation*. The proponents of statistical ensemble interpretation tend to assert that it is possible to have simultaneous knowledge of x and p_x with good resolution. One way of doing this, for a free particle, would be first to measure p_x, assume that this variable is conserved (since it is a "non-demolition" variable), and then measure x. Using the hypothesis that the position measurement is faithful (see previous section, item 2), one would have simultaneously well-defined values for x and p_x, right before the second measurement! In this way, according to this interpretation, the uncertainty principle would not prohibit the existence of simultaneous well-defined values for a same particle. What happens (following the argument of Margenau, 1937, p. 361) is that if one prepares the same quantum state $|\psi\rangle$ many times, and measures p_x and x for each preparation, then one would obtain values that vary from one measurement to the other. If these values are put in a histogram for x and p_x, one obtains the standard deviations Δx and Δp_x. Therefore, the uncertainty principle would be an exclusively statistical thesis, contrary to the claim of the other interpretations (see also Ballentine, 1970).

(3) *Realist Dualist Interpretation*. According to this view, the particle always has simultaneously well-defined x and p_x but these values are unknown. If we measure x with good *resolution*, we necessarily have a large *uncertainty* or ignorance of p_x, because the measurement of x by a macroscopic apparatus disturbs in an uncontrollable way the value of p_x. With respect to the uncertainty principle, this interpretation is quite close to the corpuscular view seen above.

(4) *Positivist Dualist Interpretation*. We have seen that a phenomenon cannot be corpucular and undulatory at the same time. In an analogous way, it would be impossible to measure simultaneously x and p_x with resolutions smaller than Δx and Δp_x given by the uncertainty relation. Curiously, the original argument given by Heisenberg to justify the uncertainty relations, by means of a gamma ray microscope, may be classified in interpretations 2 or 3 (being for this reason sometimes called a "semi-classical" argument). But since he shared a *positivist* thesis, according to which only that which is observable has reality, he could conclude in this case (after the determination of position) that "it is meaningless" to speak of a particle with well-defined momentum.

4. The Main Interpretations of Quantum Theory

We have divided the interpretations of quantum theory into four broad groups, according to two criteria: (i) *Ontology*: what is the ultimate nature of physical reality? Particles, waves, or some kind of dualism? (ii) *Epistemology*: to what extent does the theory describe this reality? Does it only describe the reality that can be observed and measured (positivism) or does its theoretical concepts also correctly represent (or attempt to represent) a reality which stands beyond observation (realism)?

The four groups of interpretations obtained were: (1) Wave (undulatory), (2) Particle (corpuscular), (3) Realist Dualist, and (4) Positivist Dualist. The wave and the particle interpretations tend to be realist, but they also come in more positivist versions, and the transition between the different groups is quite smooth, as we will see. Let us start by comparing the division presented here with the usual classifications of interpretations.

In the chapters of his celebrated book on the philosophy of quantum mechanics, Max Jammer (1974) presents five groups of interpretations: (*i*) the semi-classical pioneering views, (*ii*) the complementarity conception, (*iii*) hidden variables theories, (*iv*) stochastic views, and (*v*) the statistical ones. One may also add a further group, suggested by Redhead (1987, ch. 2) and others: (*vi*) potentiality interpretations.

4.1 The First Semi-Classical Theories

The pioneering *semi-classical* theories considered by Jammer are realist interpretations that appeared between 1926-27. They consist basically of what we have called wave and dualist interpretations. The undulatory ones include the initial electromagnetic view of E. Schrödinger (1926) and the hydrodynamical interpretation of E. Madelung (1926), the latter being later developed by other physicists, including the Brazilian Mário Schönberg (1954). One of the dualist views was the pilot-wave theory of L. de Broglie (1926), abandoned in the following year and reabilitated in 1952.

Among the semi-classical theories, Jammer also includes the initial probabilistic interpretation of Max Born (1926), according to which $|\psi(r)|^2$ expresses the *probability* of finding *one classical particle* in a certain region. In order to explain interference phenomena, such a particle would be accompanied by a "ghost field" (term used by Einstein), a "probability wave" which would propagate in space. This renders the view dualist, although Jammer preferred to consider it corpuscular.

Subsequently, this interpretation of Born was weakened, and $|\psi(r)|^2$ became the probability of *measuring* a quantum by means of a detector located in a certain region. Since this thesis was incorporated into the minimum formalism of quantum theory, we shall call it "Born's rule" (and not "Born's probabilistic interpretation"). Strictly speaking, Born's rule shouldn't even refer to "probability", but rather to "relative frequency", which is the directly observable datum in the empirical basis. To consider that the relative frequency is a "probability" is, strictly speaking, an *interpretation* of the formalism. Accepting this interpretation of quantum theory (as is usual), one arrives at different views of the quantum world, according to the interpretation adopted for the notion of probability (within the theory of probabilities).

4.2 The Complementarity Interpretation

The interpretation taken to be the most widespread among physicists is the *complementarity interpretation* developed by Niels Bohr in the years 1927-35, the theses of which were presented above as representing the positivist dualism. It is also known as the Copenhagen interpretation, refering to Bohr's hometown and where Heisenberg worked at the time, and also where Pauli met them in June, 1927, to reconcile their divergent opinions. Heisenberg had written his famous paper on the uncertainty principle, emphasizing a corpuscular perspective. Bohr, who had developed his idea of complementarity during a skiing trip to Norway, in March, found several mistakes in the paper, and emphasized that both a wave and a particle picture were necessary to derive the uncertainty principle. Pauli and Bohr succeeded in convincing Heisenberg that complementarity was consistent with the uncertainty principle, and thus was born the new interpretation that soon would become consensual in the community of physicists, leaving behind the semi-classical views mentioned in the previous section.

The *principle of complementarty* claims that an experiment can be represented either in a corpuscular picture, or in an undulatory picture, according to the situation. To say that such representations are complementary means that they are mutually exclusive, but together they exhaust the description of the atomic object. An experiment is in accordance with a corpuscular representation if it is possible to infer the past trajectories of the detected quanta. It is in accordance with a wave representation if it presents an interference pattern. It is an empirical thesis (that is, a thesis the acceptance of which does not depend on the adopted interpretation) that a same experimental setup cannot exhibit both clear interference patterns and unambiguous trajectories (see Pessoa 1998).

Why wouldn't it be possible to encompass a quantum object in a more general single picture? Because, according to Bohr, we are limited by the language of classical physics, the language we use to communicate to others how an experimental arrangement is set up and what are the results of measurements taken, the language that describes the macroscopic world. We can only have access to the quantum world by means of apparatus describable in classical language. Would this imply *macrorealism*, that is, the thesis that macroscopic objects (like Schrödinger's cat) cannot exhibit quantum properties? Not necessarily: what Bohr defends is that it is always necessary to use a classical apparatus to measure quantum properties, but parts of this apparatus may be treated as a quantum system.

As mentioned in section 3.1, Bohr's starting point was the "quantum postulate", which attributes to any atomic process an "essential discontinuity" or "individuality". According to Bohr, one consequence of this is the impossibility of controlling or predicting the disturbances arising in the quantum object due to the interaction with the measurement apparatus.

In 1935, Einstein, Podolsky & Rosen (EPR) published their famous article in which they argued that quantum mechanics is an incomplete theory (a thesis shared by the statistical ensemble interpretation). The argument involved a pair of correlated particles located at a distance from each other. Assuming that the measurement operations in one of the particles could not instantaneously affect the other particle (the thesis of *locality*), they concluded that there would be elements of reality which quantum theory could not describe (and in this sense it would be incomplete).

To answer EPR, Bohr had to refine his interpretation, giving emphasis to the *wholeness* which encompasses the experimental setup and the quantum object, and coining the term "phenomenon" to refer to an instance of this wholeness. Thus, even if an apparatus has parts which are separated at a great distance, a change in one of these parts would modify the wholeness of the phenomenon, modifying the elements of reality. There would therefore not be elements of reality not describable by quantum mechanics: the theory would be complete. The essence of Bohr's argument seems to have been the (not very explicit) rejection of the notion of locality of Einstein, with his conception of wholeness (see Bohr 1949). The change of a distant part of the apparatus followed by a measurement would result in an instantaneous modification of the overall wave function. However, since the wave function does not refer to reality (according to this interpretation), this would not violate in an explicit way the assumption of locality (only in 1952, with Bohm, would such an assumption be explicitely quetioned).

Thus, in his answer to EPR, Bohr gave priority to the wholeness involving apparatus and object, resulting in a "relationalist" conception, according to which the quantum state is defined by the relation between the quantum object and the whole measurement apparatus.

In section 4.7, we will survey these and other opinions of the founders of quantum mechanics, which form the group of views which constitute the *orthodox interpretations*.

4.3 Hidden Variable Theories

Hidden variable theories are proposals that introduce additional parameters to quantum theory. Such parameters are not directly observable, but their values are taken to determine in a unique way the result of a measurement and, on the average, they furnish the expected values of quantum mechanics. According to Jammer, the Russian J.I. Frenkel, Born's assistent, sketched an interpretation of this kind in 1926. In 1932, von Neumann presented his famous proof of the impossibility of hidden variables, but such a proof did not encompass all the possible types of hidden variables theories, as J.S. Bell would show clearly only in 1966. Von Neumann's proof did not consider, besides other things, the possibility that the hidden variables belong to the measuring apparatus.

This was the property (called *contextualism*) that rendered possible the realist dualist interpretation of David Bohm (1952). Writing the wave function as $\psi(x) = R(x) \exp[iS(x)/h]$, where S and R are real functions, Bohm assumed that $\psi(x)$ described an ensemble of particles with position $\boldsymbol{x}$ and momentum given by $\boldsymbol{p} = \nabla S(\boldsymbol{x})$. Position and momentum would thus be the hidden variables of his interpretation. He then obtained the Newtonian equation of motion, $m\boldsymbol{a} = -\nabla V(x)$, where the $V(x)$ is the sum of the classical potential function and the *quantum potential* $U(x)$, which has the following form: $U(x) = -(\hbar^2/2m)\,\nabla^2 R(\boldsymbol{x})/R(\boldsymbol{x})$. Note that even if the absolute value R of the wave function has a small value (corresponding to a distant tail of ψ), the potential might have a significant value (since R appears both in the numerator and in the denominator). The potential $U(x)$, which expresses the undulatory aspect of the model, has the property of "non-locality" (that is, it acts in an instantaneous way even at a long distance), besides not having a definite source. More recently, there has been an interest in "Bohmian mechanics", but the quantum potential has been treated in a non-realist manner, as an unnecessary hypothesis (see Cushing et al., 1996).

The *pilot wave interpretação* proposed by L. de Broglie in 1926-27 is formally similar to Bohm's for a single particle, but differs for more particles. For de Broglie, the particle is considered a "singularity" of its own field ψ (behaving like a soliton), and the waves of this field propagate in the physical 3-dimensional space, and not in configuration space, as for Bohm. One experimental consequence of this interpretation has been proposed by Croca et al. in 1990, but its prediction has been refuted by Wang, Zou & Mandel (1991), which falsified the pilot wave interpretation in 3-dimesnional space.

Interpretations that introduce hidden variables may be corpuscular, undulatory or dualist, or they lack any physical interpretation. The Bohm & Bub (1966) theory, for example, introduces an additional Hilbert space (without a physical interpretation), in such a way that the vector in this space (distributed in a random manner) is the hidden variable (see Belinfante, 1973).

4.4 Stochastic Interpretations

Stochastic interpretations are hidden variable theories that are inspired by the theory of Brownian motion, and by the fact that the Schrödinger equation is formally identical to a diffusion equation with an imaginary coefficient. Such theories are essentially classicist, are usually corpuscular and attempt to be local. For F. Bopp (1954), the matter waves of quantum physics are the result of the collective motion of submicroscopic particles (as in the case of sound). More recently, the so-called "stochastic electrodynamics" has retained the corpuscular ontology for particles with mass, but considers light as a classical wave with boundary conditions which include fluctuations in the electromagntic vaccum (Boyer, 1975). Usually such interpretations are able to derive the Schrödinger equation, but have difficulty in explaining the measurement process (see survey in Ghirardi et al., 1978).

4.5 Statistical Ensemble Interpretation

The *statistical ensemble* interpretations (or simply "statistical interpretations") assert that the wave function does not refer to an individual system, but to an ensemble of systems prepared in a similar way. The American physicists J. Slater (1929) and E. Kemble (1937) defended such views, which became quite popular in the Soviet Union (Blokhintsev), as a rection against the subjectivism of the orthodox interpretations. K. Popper, H. Margenau and A. Landé are other thinkers associated with this view, and the latter declared explicitly: "Particles, yes! Waves, no!". The notion of "wave-particle duality", and also that of "collapse of the wave packet", are rejected by this corpuscularist view.

L. Ballentine, in an influential paper published in1970, defended that the statistical ensemble interpretation does not have to commit itself to an ontology, which led to the distinction between: (*i*) a *"minimal" ensemble interpretation*, which adds to the minimal formalism of the theory only the thesis that the state represents an ensemble, leaving open the question of the nature of the elements of this ensemble; (*ii*) and an interpretation involving hidden variables, usually corpuscular, which is sometimes called *ensemble interpretion with intrinsic values*. The latter is clearly realist, while the former is more positivistic (for an example of a positivist ensemble interpretation, see Park, 1973).

Maybe the most attractive aspect of the ensemble interpretation is its analysis of the uncertainty principle, presented in section 3.6.

The greatest difficulty of any corpuscular view is explaining interference experiments. Landé (1965-75) argued that this explanation can be based on an old proposal of W. Duane (1923), according to which there is a discrete transfer of momentum from the crystalline lattice (which causes diffraction) to the particle (which is diffracted). Such an explanation, however, does not work for interference experiments which do not involve rigid lattices, such as the electron biprism (as pointed out by Rosa, 1979; see also Home & Whitaker, 1992).

4.6 Potentiality Interpretations

Michael Redhead (1987) has classified the interpretations of quantum mechanics into three mais groups, according to the answer given to the following question (compare with section 3.5): what can be said about the value of an observable Q, when the system is not in an eigenstate of the corresponding operatir? (View A:) Hidden variable theories claim that Q has a well-defined but unknown value. (View C:) Complementarity asserts that the value of Q is not defined or is "meaningless". (View B:) The last group suggests that Q has a not well-defined value, a diffused, smeared out or fuzzy value.

What this latter view B proposes, according to Redhead, is that, in reality, the system does not possess well-defined values, but propensities or *potentialities* for producing different measurement results. This Aristotelian notion, of potentialities that are actualized during measurement, appear in the writings of Heisenberg in the 1950's, which may be classed as an orthodox interpretation. The idea is also formulated Margenau (1954), with his "latent" magnitudes (ensemble interpretation). Redhead concludes that this is a realist view.

Such notion of potentiality or intermediary reality can also be attributed to the interpretations that we have called “undulatory”. It wil be argued, in section 4.8, that this is an important class of interpretations, although books like Jammer (1974) tend to omit this group (Jammer describes some of these interpretations in different chapters of his book). The notion of potentiality is also close to the “implicate order” recently proposed by David Bohm

It is curious that different classes of interpretations (which we have called corpuscular, wave and positivist dualist) make use of this notion of potentiality or potential reality.

4.7 The Orthodox Interpretations

We will now survey the slight differences that exist between different interpretations usually classified as “orthodox”. Generally, they have a commitment with *dualism*, but the boundaries with corpuscular interpretations, on one side, and wave interpretations, on the other, are rather diffuse. Most of them also present a positivistic attitude, but once again the boundary with realist dualism is smooth.

(a) *Complementarity Interpretation*. This is the “Copenhagen interpretation” defended by Bohr since 1927, and with a greater emphasis on relationism since 1935 (see section 4.2). Pauli and Rosenfeld kept very close to this view, while Heisenberg and Born stood a little farther away. The positivist stance is expressed by the impossibility of attributing a tyope of phenomenon (wave or particle) to an experiment before the measurement is completed. However, after the measurement is completed, Bohr accepted the use of retrodiction.

(b) *Postivist Wave Interpretation*. This term refers to the position assumed by von Neumann (1932), by Wigner (1963), and by many theoretical physicists. Emphasis is given to the state vector $|\psi\rangle$, that is reduced (collapses) after measurements; even the measurement apparatus is described by a state vector. This position is sometimes called the “Princeton interpretation”. It does not explicitly attribute reality to $|\psi\rangle$ (in this sense, it is positivist), but the calculations are as if $|\psi\rangle$ corresponded to a reality.

(c) *Subjectivist Interpretation*. This is the approach adopted by London & Bauer (1939), occasionally defended by Wigner (1962) and some others (like Jeans, Eddington, and Heitler), and which reappeared in the 1990’s (for example, with H. Stapp). Adopting an undulatory view, it argues that human consciousness is responsible for the collapse. In the words of London & Bauer: “the irreversible transformation in the state of the measured object” would be due to the “faculty of introspection” or to the “immanent knowledge” that the conscious observer has of his own state. This position is a development of view (b), while $|\psi\rangle$ may be treated as a real entity. In this case, it is not a positivist (descriptivist) view, but an idealist one, in the sense that the reality described by quantum mechanics depends on the presence of a human observer.

(d) *Macrorealist Complementarity Interpretation*. The Russian school that defended complementarity (Fock, 1957, and Landau, according to Bell, 1990, section 6) did not accept the position of Bohr and von Neumann, according to which the boundary between the classical and quantum worlds could be drawn at any point in the chain connecting the object to the *observer* (“psychophysical parallelism”). In a more objective way, this Russian school attributed attributed classical propoerties to macroscopic objects in general. A similar position was proposed by Ludwig (1961), who postulated that non-linear corrections to the Schrödinger equation would impose classical behavior for macroscopic bodies.

(e) *“Eclectic” Interpretation.* Jammer (1974, p. 68) attributes to Heisenberg the following position, in the beginning of 1927: both an exclusively corpuscular and an exclusively undulatory interpretation could be associated to the quantum mechanical formalism. In 1930, Heisenberg still thought along these lines, but stressed that each representation had its limitations. This eclecticism is sometimes adopted in quantumfield theory to explain both the success of Feynman's corpuscular view and of Schwinger's undulatory approach.

(f) *Realist readings of Complementarity*. This is a path to be explored in the future. In 1927-28, Bohr presented the principle of complementarity by opposing "definition" (a pure state of a closed system) e "observation" (a measurement renders the system open and intorduces indeterminism). However, he dropped this caracterization since it did not make sense for positivism to refer to a non-observed system. Realist readings, however, make pick up this type of complementarity again. David Bohm, in his text book of 1951, also made a more realist reading of complementarity (failing in a few points), stressing that unpredictability is connected to the coupling of the quantum object to the Universe as a whole (during measurement). In another direction, realist readings of complementarity lead to paradoxical situations, such as the assertion that "the photon knows what will be the future experimental setup", which helps to increase the mystery of quantum theory for the larger public. John Wheeler makes this kind of realist reading, concluding (in the delayed-choice experiment, due to retrodiction) that "the past does not have existence while it is not registered in the present" (Wheeler 1983, p. 194).

(g) *Radical Instrumentalism*. In a review of possible interpretations of the measurement problem, Wigner (1983) mentioned the view according to which the aim of quantum mechanics would not be to describe reality, but to furnish statistical correlations between successive observations. This "instrumentalist" point of view is quite common among physicists, radicalizing the psotitivism of the orthodox interpretation and the epistemic view of the quantum state. J. Park (1973), a former student of Margenau, arrived at this position from the statsitical ensemble interpretation: "Quantum Mechanics is a theory about the statistics of measurement results".

(h) *Stroboscopic Interpretation*. Within the latter radical view, one may place a *stroboscopic* corpuscular interpretation, according to which the particles in nature give discontinuous leaps from one position to the other, according for example to the macroscopic track left by the particles in a Wilson cloud chamber. Heisenberg (1927, p. 63) discusses this possibility, stressing that in this case an instantaneous velocity is not well defined (see also Bohm, 1951, p. 144-8).

(i) *S-Matrix Interpretation*. Another instrumentalist version is the interpretation given by the S-matrix theory. This approach describes scattering processes by considering only the assintotic initial and final states, and S-matrix which relates one to the other. Under certain conditions, one can show that this approach is identical to the use of the Schrödinger equation, having however the advantage of being easily extended to the relativistic domain (Stapp, 1971).

(j) *Sum over Histories Interpretation*. In 1948, Feynman presented his "sum over histories" approach, developed in relativistic quantum field theory, as a new interpretation for quantum theory. A particle would follow all possible paths, and the wave function would be a sum of these amplitudes (histories). This approach stresses a corpuscular representation, but it is worth investigating to what point it is not an undulatory view.

4.8 Wave Interpretations

The wave (or undulatory) interpretations consider that the quantum state corresponds to some kind of reality (in opposition to the orthodox views), and deny the existence of point particles that follow continuous trajectories. Thus, in agreement with the complementerity interpretation, and contrary to the ensemble, stochastic and realist dualist interpretations, they accept that the description by means of the quantum state is complete, and that systems prepared in the same state are in fact identical.

Max Born, on a certain occasion, defended the reality of $|\psi\rangle$ when he wrote: "I personally like to consider a probability wave, even in 3N-dimensional space, as a real thing, as certainly more than an instrument for mathematical calculations. Because it has the feature of an invariant of observation" (Born, 1949, pp. 105-6). In opposition to this, but for the same reason, Heisenberg (1958, p. 129) prefers to consider the ψ wave as something "objective", but not "real".

In the last decades, there has been an increase in the number of interpretative proposals that are akin to the undulatory view, assuming that the wave function corresponds to a reality. A positivist argument used against this view is that one cannot atribute reality ψ because it would be impossible to determine the quantum state from a single measurement. Attempting to refute this argument, Aharonov et al. (1993) proposed a new class of measurements, called "protective", that would allow the detremination of the quantum state. Such a proposal, however, has been much criticized.

Let us now give an overview of the tradition of wave interpretations, the unity of which has received little attention (one member has already been examined in section 4.9c).

(a) *Electromagnetic Interpretation*. In Schrödinger's original proposal (mentioned in section 4.1.), $e|\langle \psi_i|\psi\rangle|^2$ represented a classical charge density (where e is the system's total charge), so that one would have "matter waves" and not "probability waves". Such waves would propagate in a deterministic way, recovering classical visualization. Particles would be, in reality, wave packets.

The arguments presented at the time, which undermined this proposal were: (*i*) *High dimensionality of* ψ. For N particles, $|\psi\rangle$ is defined in 3N-dimensonal configuration space. How could this be interpreted? (*ii*) *Particles as wave packets*. Wave packets disperse as time goes by, contrary to what happens in the special case examined by Schrödinger, in the quantum-mechanical harmonic oscillator. (*iii*) *Discreteness of atomic processes*. How to explain quantum leaps, charge quantization, and how to associate discrete atomic frequencies to discrete energies ($E=h\nu$)? (*iv*) *State reduction during measurement*. How to explain the apparent state collapse that occurs during measurments, expressed by the projection postulate, and the non-locality that is involved?

More recently, some authors have reexamined Schrödinger's original proposal, offering solutions to above mentioned problems (Dorling, 1987; Barut 1988). Some of these solutions will be mentioned below.

(b) *Hydrodynamical Interpretation*. Starting from Schrödinger's equation and writing $\langle \psi_i|\psi\rangle = \alpha\, e^{i\beta}$, Madelung (1926) obtained a hydrodynamical equation for α, thus suggesting that a fluid with distributed charge and mass composes the basic structure of the world. Such an approach would be reconsidered by Bohm (1952), who added however a particle. Bohm & Vigier (1954) presented a hydrodynamical model in which the fluid is coupled to stochastic fluctuations at a subquantal level (see Jammer, 1974, pp. 33-8, 49-54).

(c) *Naïve Wave Interpretation with Collapses*. A realist undulatory view may be obtained by adapting von Neumann's positivist interpretation (section 4.7b). In this case, collapses would be real processes, the causes of which could be associated to resonances due to the interaction of the apparatus with the environment, or simply accepted in an *ad hoc* way. Non-locality would be present both in the process of collapse and in measurements of correlated particles associated to Bell's theorem.

(d) *Relative States Interpretation*. In 1957, H. Everett postulated that the universe as a whole could be described by a single wave function that evolves deterministically, according to Schrödinger's equation. The apparent collapse associated to measurements would, in reality, be an illusion, linked to the fact that our brain is also coupled to quantum objects. The brain would participate in a superposition of states associated to different readings of the measurment results, and each one of these "memory configurations" would not have access to the others.

The world would branch in this way in many parallel worlds during each act of measurment. In spite of the apparent absurdity of this interpretation, it raised a lot of interest around 1970 (DeWitt, 1970), and today it has generated once again much discussion, with David Albert and others, in different variations: many worlds, many minds, and bare theory (see Barrett 1999).

(e) *Wave Interpretation with Decoherence*. The "decoherence" approach attempts to explain the emergence of classical behavior in a quantum system (for example, after measurements) from the interaction between obejct, apparatus and environment. Authors like Zurek have initially placed themselves closer to the complementarity interpretation, while others like Zeh & Joos adopted an undulatory view. Zeh (1993) has asserted that "There are no quantum leaps, and there are no particles!". The approach of these authors offers a solution to problem (*ii*) mentioned in entry (*a*) above: as a free wave packet disperses, collisions with other particles induce a "localization" of the system (which however ceases to be in a pure state).

(f) *Spontaneous Localization Interpretation*. Ghirardi et al. (1987) and also Gisin & Percival (1992) have attributed reality to the wave function, but assume that the process of collapse (for a wave packet narrowly centered around a certain position) is spontaneous or stochastic (which places this approach also within the stochastic interpretations). In order to eliminate subjectivism, they suppose that all the particles have a very small probability of suffering a localization, which would not affect the validity of Schrödinger's equation for few particles. In the case, however, in which a macroscopic object couples to a measurement apparatus with octillions of particles, the probability of localization becomes very large, thus explaining the state reduction that accompanies direct measurments of position.

(g) *Transactional Interpretation*. This approach is based on the "transaction" between an emitter and an absorber, which takes place by means of retarded waves (the usual ones) and advanced waves (which propagate with negative energy towards the past), according to the proposal of Wheeler & Feynman (1945). This interpretation of quantum mechanics developed by Cramer (1986) is temporally symmetric, non-local and considers that the wave function is a wave in 3-dimensional space.

4.9 Interpretations that question Classical Logic

In this section we group some views that propose modifications of classical logic in order to explain the interpretative problems of quantum mechanics. What they have in common, besides questioning different aspects of clasical logic, is a certain sympathy for the attribution of well-defined values for all observables, which puts them close corpuscular views or to hidden variables theories.

(a) *Quantum Logic*. Since the pioneering work of G. Birkhoff & von Neumann (1936), it is common to assert that the logic of the microscopic world is of a special type, called "non-distributive logic" (see for instance Hughes, 1981). Such a conclusion is defensible, but it presupposes a corpuscular interpretation (dispersion-free values) for quantum theory.

(b) *Operational Approach*. A certain approach to quantum logic (that does not assume a corpusuclar ontology) considers the theory not as a description of physical nature, but a description but as a description of the behavior of the scientist while he prepares and measures microscopic objects in the (Foulis & Randall, 1974).

(c) *Modal Interpretação*. In a broad sense, this name applies to any interpretation that is inspired by modal logic, which makes use of the categories of "possibility" and "necessity". More specifically, it refers to the interpretation proposed by Kochen (1985), which considers the problem of what are the *properties* (that is, what are the observables with well-defined values) of a subsystem that is quantically correlated with another (making use of Schmidt's decomposition theorem). Such a relational realism (the properties exist in relation to the chosen environment) furnishes an explanation to the EPR paradox without assuming non-locality.

(d) *Consistent Histories*. A "history" is a series of well-defined properties occurring in an temporally ordered sequence (for exemple, $p_x(t_1)$, $x(t_2)$, $p_x(t_3)$). In 1984, R. Griffiths introduced the notion of "family of consistent histories", to which one may attribute a probability to each history. Given an initial event D and a final event F, this approach furnishes a probability for the occurrence of a history of intermediary events E_1, E_2, etc. If the initial event D is $S_x=+½$ (after the measurement of spin in the x direction) and the final F is $S_z=+½$ (after a measurement of spin in the z direction), the probability of an intermediary event E being $S_x=+½$ is 1, and the probability of being $S_z=+½$ is also 1! However, since these two histories are not consistent, one cannot deduce that $S_x=+½$ *and* $S_z=+½$ with probability 1, for the same event E. This violates classical probability calculus (see criticisms of d'Espagnat, 1989).

Other authors, like Omnès, Gell-Mann and Hartle, worked on this interpretation proposing that it is a development of the orthodox interpretation, since the latter only attributes probabilities to the moment of measurement, while the consistent histories interpretation would allow attributing probabilites to past events. Omnès (1992) even defended the use of what he called a "quantum logic", but it is simply an approximation rule that eliminates very small quantities. Implicit in Griffiths' approach is the acceptance of retrodiction, of the epistemic view of states, and of faithful measurements. His view is clearly dualist, since retrodiction may also lead to states involving superposition of trajectories.

5. Map of the Interpretations

Now that we have become familiar with many interpretations of quantum theory, let us make a sketch of the position of each of them in relation to the *ontological* (particle, wave, dualism or without ontology) and *epistemological* (realism or positivism) criteria. In the map of Fig. 1, the horizontal axis presents the ontological criteria, while the vertical axis is divided into realism (on the bottom) and positivism. Certain regions are highlighted, corresponding to the following interpretations: orthodox (ORTH.), statistical ensemble (ENS.), hidden variables theories (HVT.), wave (WAVE), stochastic (STOCH.), and quantum logics (LOG.). In general, the hidden variables theories may be considered a particular case of the ensemble interpretation. Interpretations that are related appear with a dashed line between them.

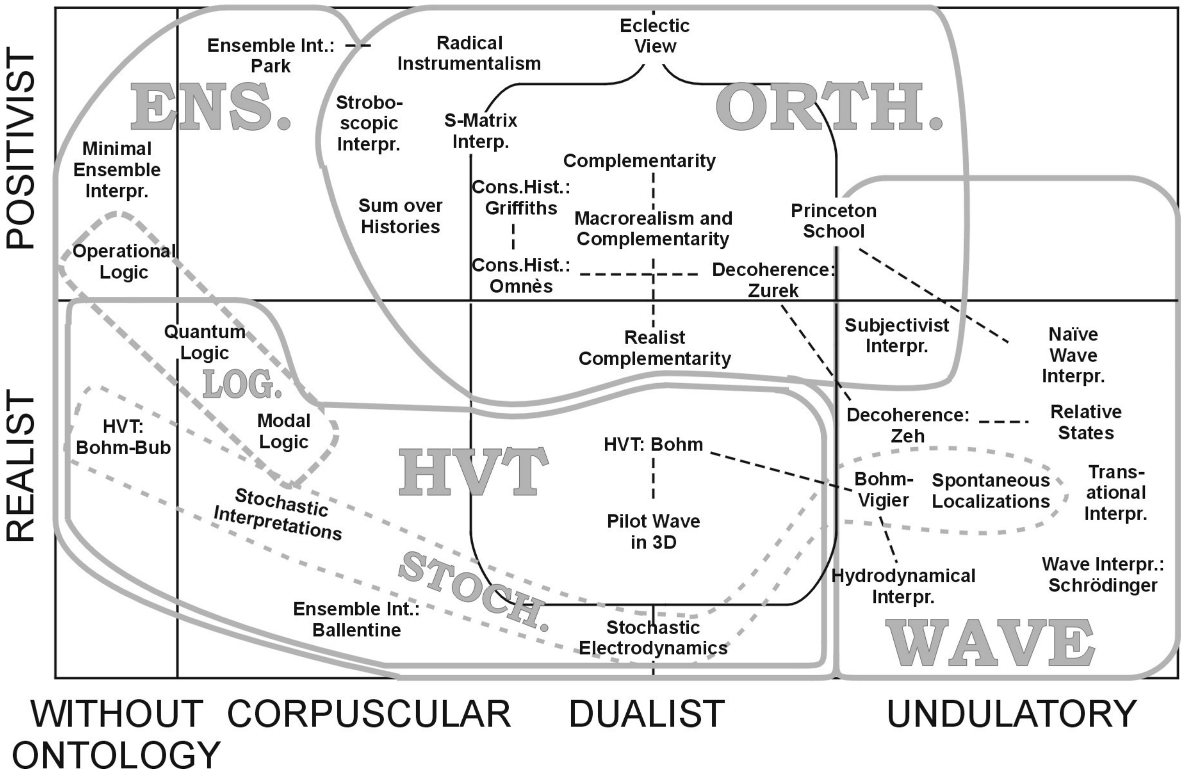

Figure 1: Map of interpretations of quantum theory.

6. Conclusion

The *systematic* study of the interpretations of quantum theory is still a vast and not much explored field. It is the task of "philosophy of physics" to try to systematize the comparative study of interpretations, pointing out which theses each view answers in a clear way, which assertions in fact correspond to a specific ontology and which are only the attribution of a label, which problems are sweeped under the rug, and how to group the interpretations in a satisfactory way. Furthermore, it would be interesting to take into account the intentional-emotional aspects mentioned in section 1, and extend the study not only to the "declared" interpretation, but also to the "natural" interpretations of alternative formalisms (like the Wigner distribution).

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