Complementing the Principle of Complementarity

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Abstract

This article explores various aspects of the principle of complementarity in quantum physics, and certain proposals for refining and extending it are suggested. After an introduction to wave-particle duality, retrodiction, and Bohr's conceptions of complementarity, we analyze the double-slit experiment for a pair of correlated particles and survey the discussion on the ultimate explanation for the principle of complementarity. Proposals for refining the principle are given while we explore variations of the Mach–Zehnder interferometer. Four kinds of "modifications that preserve the type of the phenomenon" are presented, which justify calling a phenomenon "corpuscular" even when trajectories cannot be inferred. We point out that the term "phenomenon," which refers to the object's state and to the disposition of the apparatus, should also refer to the particular quantum being detected, while the specific nature of the apparatus is irrelevant for judging a phenomenon. The last part of the paper presents a formal definition of "phenomenon" as an orthonormal basis obtained by "retrodiction with certainty," and "complementary phenomena" as those associated with "mutually unbiased bases." We then address the question of whether complementarity only involves pairs, and examine two alternative solutions: the formal definition claims that there are $N + 1$ mutually complementary phenomena (for an N-dimensional space), while Bohr's exhaustive definition associates the same "wave aspect" with all phenomena that are mutually unbiased with respect to the corpuscular one. Intermediate phenomena are briefly analyzed in the case of a beam splitter with variable transmittance, and shown to be problematic for the exhaustive definition of complementarity. We thus conclude by presenting a definition of complementarity that favors the formal approach.

Key words: quantum mechanics, complementarity, retrodiction, wave-particle duality, Mach–Zehnder interferometer, intermediate phenomenon

1. INTRODUCING THE PRINCIPLE OF COMPLEMENTARITY

The idea of *complementarity* was introduced by Niels Bohr in 1927, constituting the basis of his interpretation of quantum mechanics, which by 1935 acquired its final form. Although there are at least three basic types of complementarity in quantum physics (besides versions in other branches of science), the name is most often attached to the wave-particle duality, to what may be called the "complementarity between experimental setups." Anyone with a degree in physics has been taught that an electron may appear either as a particle or as a wave, but never as both simultaneously. That is the main idea behind the principle of complementarity, which we will express in the following loose form, to be refined as this paper goes along:

C_1. A specific quantum-mechanical experiment may be represented either in a *corpuscular* picture or in a *wave* picture (never both). These pictures are *complementary* aspects of experience: they are mutually *exclusive*, but only together do they *exhaust* the description of the atomic object.

2. THE WEAK VERSION OF THE WAVE-PARTICLE DUALITY

Let us now examine a concrete quantum-mechanical experiment and see how the wave and particle aspects of matter arise. We will choose the famous electron two-slit experiment, which is most easily realized by means of a setup known as an "electron biprism."[1] A beam of electrons is diffracted through a single slit O, and passes around a positively charged wire (Fig. 1). The amplitudes that pass on each side of the wire are deflected toward the other side. What is observed is a nice-looking interference pattern in the region R where the two amplitudes superpose.

In the 1980s, physicists started being able to observe the individual effects of *single* interfering photons, neutrons, and electrons. If each individual electron that passes through the biprism is detected, how does the interference pattern emerge? Anyone familiar with the basics of quantum mechanics knows that the electrons are detected as localized quanta (Fig. 2). As the "points" in the detecting screen build up, the interference pattern emerges.[2]

Is this a wave phenomenon, a particle phenomenon, or both? Well, our first reaction might be to say that it is *both,*

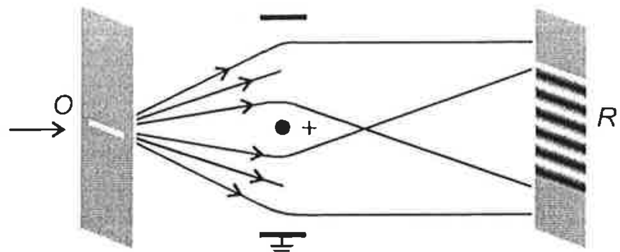

Figure 1. The electron biprism setup.

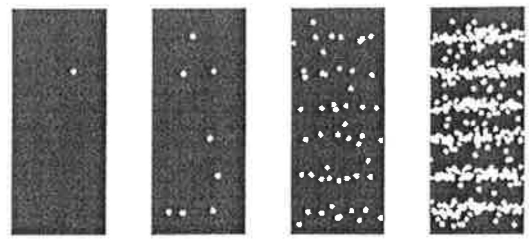

Figure 2. Buildup of an interference pattern.

since there is an interference pattern (wave aspect) together with localized individual detection (particle aspect). This is often considered a manifestation of wave-particle duality,[3] so we might call it the "weak version" of the wave-particle duality. But the definition of complementarity given in the previous section asserts that for a given experiment one has *either* the wave aspect *or* the particle aspect, not both!

What would Bohr say about this? His answer would be straightforward: the above example is a wave phenomenon. The appearance of localized points in the detection screen is due to another principle—not to complementarity, but to Planck's *quantum postulate*. For Bohr, the quantum postulate is the starting point of quantum mechanics, and it asserts that all exchanges of energy at the microscopic level take place in discrete packets or quanta, constituting the "essential discontinuity" of quantum physics. One might also add that such exchanges are always *localized.*

3. COMPLEMENTARITY OF EXPERIMENTAL SETUPS

What might be called the "strong version" of the wave-particle duality is the complementarity of experimental setups mentioned in Section 1. Let us state it in more precise terms by first noting that a *phenomenon* for Bohr (after 1935) consists of the quantum object *plus* the experimental arrangement, and, furthermore, that a phenomenon constitutes itself only after a macroscopic detection has been obtained.

A phenomenon is clearly undulatory (wave-like) if it displays some form of *interference pattern*. If one can infer a *trajectory* for the detected quantum, then it is surely corpuscular, according to Bohr. We may then give a more operational characterization of the principle as follows:

C_2. A specific "phenomenon" (quantum object + experimental setup + detection) cannot at the same time show clear interference patterns (being wave-like) and exhibit unambiguous trajectories (being particle-like).

Notice that this wave-particle principle of complementarity makes a strong empirical claim. It states that it is impossible to set up an experimental apparatus that exhibits both wave and particle aspects. In light of "intermediate" wave-particle phenomena, to be examined later on, the adjectives "clear" and "unambiguous" were introduced in the above statement. What is impossible is to have at the same time interference fringes with maximum visibility (a 100% undulatory phenomenon) and particle trajectories inferred with certainty (a 100% corpuscular phenomenon).

Putting intermediate phenomena aside for the moment, we notice that the above statement leaves room for phenomena that don't exhibit interference patterns and for which one cannot infer trajectories. Still, common usage (as in Section 1) makes reference to corpuscular phenomena even if the trajectories could only "in principle" be determined, although in practice they are not. In Section 11, we will propose a clarification of what this means.

Let us now exercise our intuition concerning the characterization of phenomena. Consider a thought experiment in which a single electron, at a known time, passes through a very small aperture (of size of the order of the electron's de Broglie wavelength) located at point O. After it passes the aperture (Fig. 3a), do we have a corpuscular or an undulatory phenomenon?

Although we tend to associate a wave picture with diffraction, the situation described above is not yet a phenomenon since the electron has not yet been detected! So suppose that a scintillation screen is inserted and the electron is detected at point R (Fig. 3b). Is the phenomenon undulatory?

No, it is corpuscular, because we can infer a straight trajectory from O to R! In fact, Heisenberg showed how diffraction from a point-like aperture may be explained within a corpuscular picture by invoking the uncertainty principle. Since the passage by the aperture may be thought of as a measurement of position x with good resolution, a large uncertainty in momentum p_x is introduced, and that explains why the electron can be detected almost anywhere in the detection screen.

On the other hand, if a double slit was inserted and the electron detected at R (Fig. 3c), the phenomenon would be undulatory. This is manifested by the interference pattern that arises after many electrons fall on the screen.

Notice that the choice of whether to insert the double slit or not may be taken *after* the electron passes by O. This is an example of a "delayed-choice experiment," much explored by John Wheeler. The choice of whether the phenomenon will be wave-like or particle-like may be delayed to an instant after the electron passes the first slit.

4. RETRODICTION

In a corpuscular phenomenon such as the one described in Fig. 3b, does the electron *really* follow a straight path from O to R? The answer given to this question will depend on the interpretation of quantum mechanics that one adopts. The inference that has been made concerning the behavior of the electron in the past is called *retrodiction*, or more specifically, "retrodiction to paths." Some interpretations make use of retrodiction, others don't. Corpuscular views (including the ensemble interpretation) tend to accept it, while wave interpretations don't. Dualist interpretations such as de Broglie's or Bohm's postulate real trajectories, but these aren't necessarily the classical-like straight paths usually conceived with retrodiction.

Within the more orthodox interpretations, Bohr and Wheeler implicitly accept retrodiction to paths when they speak of corpuscular phenomena, but both Bohr and Heisenberg stress that retrodiction is an interpretative move, which leads to no contradictions but which is "of a purely speculative character."[4]

One consequence of retrodiction to paths is that precise values for position and momentum may be ascribed to a particle in the past. In Fig. 3b, precise knowledge of the times at which the particle passed by O and fell on R allows an exact determination of its momentum vector, assuming that it propagated with constant velocity between O and R. Therefore, after the experiment is over, one may attribute well-defined position and momentum to the electron for the (past) instant of time right after the particle passed by O, in opposition to the restriction imposed by the uncertainty principle on present values of these observables.

Retrodiction to paths does *not* lead to the inference of a definite trajectory in the case of wave-like phenomena, as is illustrated in Fig. 3c. In Section 15, we will refine the definition of retrodiction in order to improve our characterization of complementarity. A philosophical exploration of Bohr's three types of complementarity is given in the Appendix.

5. DOUBLE-SLIT EXPERIMENT AND MEASUREMENT OF TRAJECTORY

Let us now consider the double-slit thought experiment for a single quantum (electron, photon, neutron, kaon, etc.), idealizing the concrete experiment examined in Section 2. Let us write out how the quantum-mechanical formalism describes the interference that is seen in this wave phenomenon.

The wave-function $\psi(r)$ that describes the state of a single electron after passing through the open slits (Fig. 4 without detector D) may be written in a simplified form as

$$\psi(r) = \frac{1}{\sqrt{2}}[\psi_A(r) + \psi_B(r)]. \tag{1}$$

To compute the probability Prob(R) of detecting the electron at point R on the screen, one should square the value of the wave-function at that point (in what follows, ψ_A should be read as $\psi_A(R)$, etc.):

$$\mathrm{Prob}(R) = |\psi(R)|^2 = \frac{1}{2}[\,|\psi_A|^2 + |\psi_B|^2 + \psi_A^*\psi_B + \psi_B^*\psi_A]. \tag{2}$$

The term $\psi_A^*\psi_B + \psi_B^*\psi_A$ is the interference term, which oscillates as R is varied.

Consider now the well-known variation in which one measures by which slit the quantum passes, with as little disturbance as possible. This may be done by inserting, after slit A, a detector D that does not absorb the particle (Fig. 4). It is an experimental fact that no interference pattern appears in the screen after many particles pass. This is readily accounted for by the principle of complementarity: if the particle is detected at A, one may infer by retrodiction the trajectory OAR, and that precludes the observation of an interference pattern.

6. DOUBLE-SLIT EXPERIMENT WITH CORRELATED PARTICLES

Let us now consider the following modification in the two-slit experiment. Instead of simply having one particle fall on the slits, suppose that this single particle is *correlated* with another particle. For example, suppose that a positron and an electron (with zero total momentum) annihilate at a certain point, generating a pair of photons. The state of this pair should be described as spherical waves in such a way that if one particle of the pair was detected at slit A of Fig. 5, the other one would necessarily have the opposite momentum, heading in the direction X_0. Analogously, a photon at B would be correlated with the other at X_1. The question now is: will there be an interference pattern at the screen (for an assembly of identically prepared particle pairs)? (The reader who is not familiar with this setup should try to answer this question before proceeding!)

The answer is that there is *no* interference pattern. This can be deduced directly from the quantum-mechanical formalism. Consider the system selected by having one of the photons pass through the slits. The state may be written as

$$\psi(r) = \frac{1}{\sqrt{2}}[\psi_A(r)\cdot|X_0\rangle + \psi_B(r)\cdot|X_1\rangle]. \tag{3}$$

Here we are using a notation that mixes wave-functions $\psi_A(R)$, $\psi_B(R)$, which refer to one particle, and state vectors $|X_0\rangle$, $|X_1\rangle$, which refer to the other particle. The mentioned

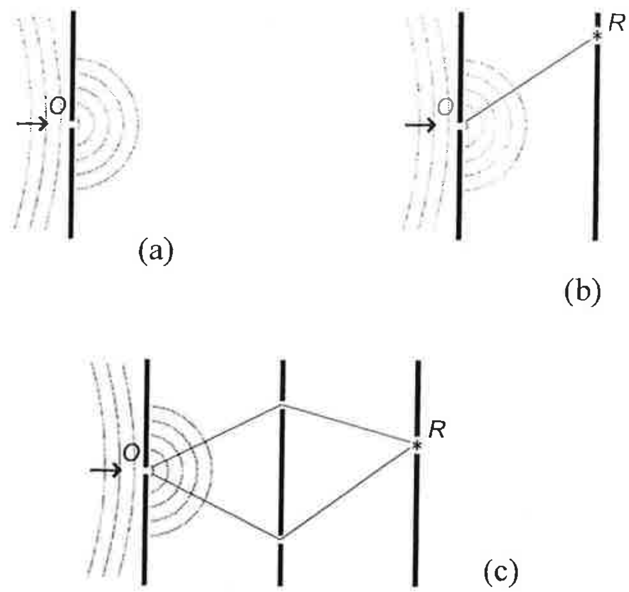

Figure 3. Different setups involving diffraction of an electron through a point-like aperture.

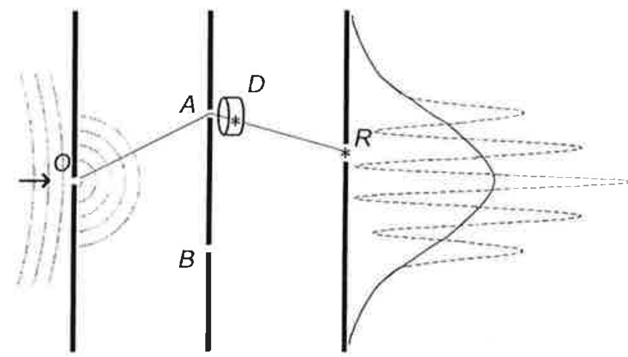

Figure 4. The interference pattern (dashed lines) for the two-slit experiment disappears (full lines) with the insertion of a detector at one of the slits.

state vectors are orthogonal: $\langle X_0|X_1\rangle = 0$. The probability of detecting a photon at point R of the screen is

$$\begin{aligned}\mathrm{Prob}(R) &= |\psi(R)|^2 \\ &= \tfrac{1}{2}[\,|\psi_A|^2\langle X_0|X_0\rangle + |\psi_B|^2\langle X_1|X_1\rangle \\ &\qquad + \psi_A^*\psi_B\langle X_0|X_1\rangle + \psi_B^*\psi_A\langle X_1|X_0\rangle] \\ &= \tfrac{1}{2}[\,|\psi_A|^2 + |\psi_B|^2\,].\end{aligned} \tag{4}$$

The interference terms vanish due to the orthogonality of $|X_0\rangle$ and $|X_1\rangle$!

The absence of interference may also be understood by means of the principle of complementarity. It is possible to measure the position of the photon heading away from the slits (to the left in Fig. 5). This measurement would indicate whether the other photon passed through the slits, and if it did, through which one (since total momentum is conserved). We would therefore be able to infer its trajectory, and the phenomenon would be corpuscular. By the principle of complementarity, no interference can be seen in this setup.

The argument in the last paragraph needs refinement. What happens if nobody measures the position of the particle heading away from the slits? Would that render the phenomenon wave-like? In this case, would the insertion of a faraway detector modify the phenomenon, according to Bohr's definition? No, that cannot happen because it would violate *macroscopic locality*: a change of apparatus on the Moon cannot instantaneously affect the statistics of outcomes here on Earth (we will return to this in Section 11). Therefore, even if nobody measures the position of the particles heading away from the slits, the phenomenon remains corpuscular, and no interference pattern will be seen in the screen.

7. EXPLAINING COMPLEMENTARITY BY MEANS OF A DISTURBANCE

We have seen how the principle of complementarity may be used to predict the qualitative behavior of experiments on quantum systems. But is it really a "principle," to be accepted dogmatically just because it helps us understand the outcome of experiments, or can it be justified in a more enlightening manner? Let us consider three classes of explanations for the complementarity of experimental arrangements. Two of them consider the *disturbance* on the quantum object arising from its interaction with the macroscopic apparatus. The third one will be postponed to the next section.

(i) *Collapse of the state vector.* Consider a wave interpretation for quantum mechanics in which the wave-function is taken to be an "objective" entity that evolves continuously until a measurement is performed on the system, leading to an instantaneous collapse of the wave-function to an eigenstate of the observable being measured. With such a view, it is easy to understand why the measurement at the slits in Fig. 4 destroys the interference pattern. The initial state after passage by the slits (1) is reduced to $\psi_A(R)$ or to $\psi_B(R)$, depending on whether the detector placed after slit A is triggered or not (assuming detectors with 100% efficiency), and it is clear that each of these reduced states by itself does not exhibit interference.

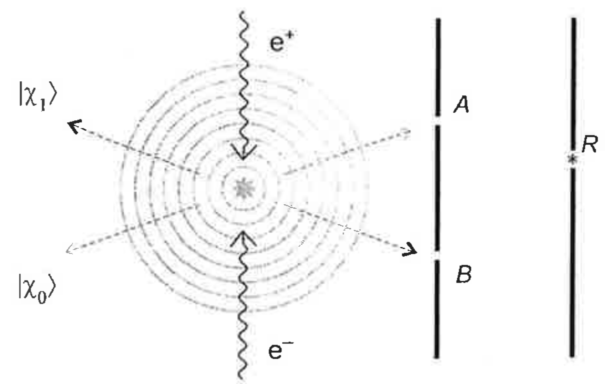

Figure 5. For a correlated pair of photons, will there be an interference pattern at the detecting screen?

The limitation of this approach is that collapse is only a sufficient condition for the washing out of interference, not a necessary one. This was suggested in Section 6, where the measurement of the second particle's position was irrelevant for the destruction of the interference pattern of the first particle. We will return to this point in Section 8.

(ii) *Decoherence by random phases, or involving the uncertainty principle.* In the 1927 Solvay Congress, Bohr's conception was challenged by Einstein, with the famous proposal of measuring the path of the electron by means of an analysis of the momentum of the plate containing the slits. Bohr(5) was able to rebut the challenge by considering that the plate is also restricted by the uncertainty relations: if its momentum was measured with excellent resolution (from which one could infer through which slit the particle passed), the position of the slits would be uncertain, and that would wash out the interference pattern. The uncertainty principle therefore can be used to explain complementarity, in this and in other setups, as we will mention later on.

An explanation that is essentially the same as the application of the uncertainty principle to the apparatus was used by Feynman and Bohm(6) to describe the effects of a measurement on the wave-function. The idea is that any nondestructive measurement alters in a random way the phase of the wave component interacting with the detector. The two amplitudes passing through slits A and B are initially coherent, but if a detector interacts with the component at slit A (Fig. 4), then the random phase that is passed to the amplitude at A results in a mutual "decoherence" of the components. This leads to a washing out of the interference pattern.

Notice that explanation (ii) does not require a state collapse. It explains decoherence (loss of interference) in an adequate way, although it cannot explain why an electron is detected in one spot on the screen and not another (to describe this, one may, according to the interpretation adopted, use either state collapse, the quantum postulate, or the existence of a particle).

8. EXPLANATION BY MEANS OF THE ORTHOGONALITY OF THE APPARATUS'S STATES

A third kind of explanation accounts for decoherence in a formal way, without explicit mention of the disturbance of the apparatus upon the quantum object.

(iii) *Orthogonality of the apparatus's states.* In the thought experiment presented in Fig. 5, the disappearance of the interference terms (4) arose directly from the quantum-mechanical formalism. Turning to Fig. 4, if we consider that the final state of the *detector* after measuring the passage of a particle is orthogonal to its initial state, then these states of the detector play analogous roles to $|X_0\rangle$ and $|X_1\rangle$ in (3) and (4). The quantum-mechanical formalism accounts for decoherence and explains complementarity in a natural way.

This idea was introduced by Scully and coworkers,(7) who suggested the following realization of the two-slit experiment, involving rubidium atoms (Fig. 6). The beam of atoms passes through two slits, is collimated, and then absorbs a quantum of light from a laser beam, making a transition to the excited state $63p_{3/2}$. Each of the components of this atomic beam enters into a "micromaser cavity," where the probability of emitting a photon (and returning to the initial ground state) is close to 1. The components then leave the cavities and are allowed to spread out in space, falling on a detection screen. If the laser beam (or, alternatively, the micromaser cavity) is turned off, one observes interference, but if the laser beam is on, the interference disappears. This is easily explained (by means of (4)) by considering that if the atom passes through slit A, a photon will be present in cavity A (even if it is not actually detected), and none in B, and this state $|1\rangle_A|0\rangle_B$ (analogous to $|X_0\rangle$) will be orthogonal to the state $|0\rangle_A|1\rangle_B$ with a photon in cavity B and none in A (analogous to $|X_1\rangle$).

The elegance and simplicity of explanation (iii), together with a calculation of the negligible uncertainty imparted "locally" by a micromaser cavity on the atom, has led Scully and coworkers to claim that complementarity would be ensured even if the uncertainty principle (explanation (ii)) did not play a role: complementarity would be more fundamental than the uncertainty principle! This bold assertion has led to an important and still ongoing debate, but the claim seems to be only partially correct. It has been argued that the cavities *do* transfer a random "momentum kick" of a certain *nonlocal* nature.(8) A question that remains is why an explanation involving disturbances (ii) is equivalent to a formal explanation (iii) not involving any explicit disturbances.

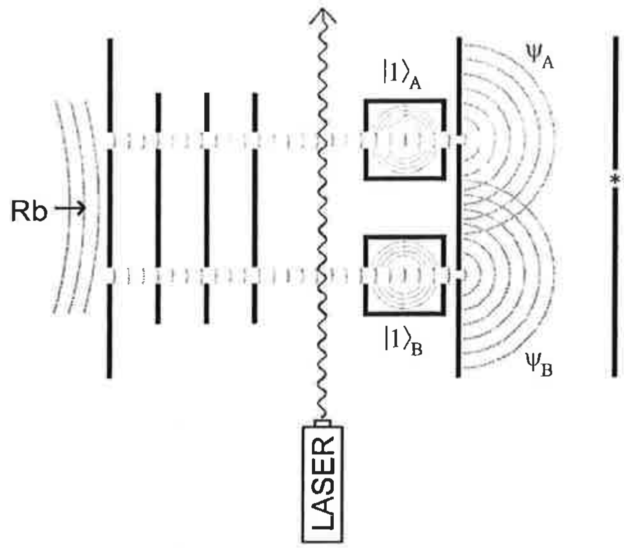

Figure 6. Atomic interference with micromaser cavities.

9. AN ADDITIONAL PUZZLE

Let us examine one more experimental puzzle, this time involving neutron interferometry. Experiments with neutron interferometry are important because they allow one to monitor the spin state of individual neutrons. In Fig. 7, a beam of neutrons (coming in from the left side) initially polarized in the spin eigenstate $+z$ is diffracted by the first "ear" of a silicon monocrystal, resulting in two component beams. Each of these falls on the second ear, resulting in four amplitudes. Two of these are lost, and two are recombined in the third ear. One of these components, however, has its spin state flipped to $-z$ by a radio frequency (rf) coil C. Interference is observed upon detection by slowly varying the phase or path length of one of the components.

The problem is the following. The experimental procedure that flips the spin state of a neutron involves the transfer of an rf photon if the neutron in fact passes through the coil. If the neutron does not pass through C, no photon is transferred. However, this difference involving one rf photon might in principle be detected in coil C. If this were possible, one would have interference together with knowledge of trajectory, violating the principle of complementarity! What is the solution to this problem?

A simple explanation[9] is that the state of the field in the rf coil is a "coherent state," one that is an eigenstate of the (non-Hermitian) annihilation operator. That would mean that the loss of a photon would leave the system in the same state! One therefore cannot infer whether the neutron passed through the coil or not merely by examining the state of the coil. An explanation in terms of the uncertainty principle between phase and particle number may also be given.[10]

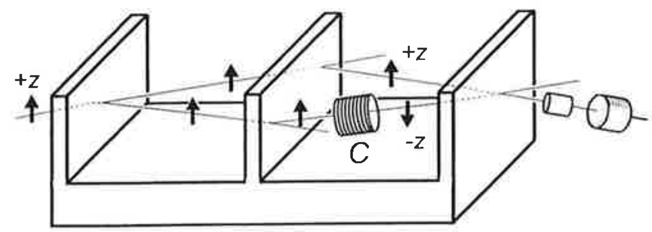

Figure 7. Neutron interferometry with a spin flipper.

Coherent states are important for defining the transition to classical systems, as noted by Glauber.[11] When a hot wire emits an electron, which passes by a small aperture in a wall, and this electron then exhibits interference, it is important that it not be in an entangled state with the hot wire or with the wall. This is ensured in the simplest way by assuming that these macroscopic systems are adequately described by coherent states.

10. EXPLORING THE MACH–ZEHNDER INTERFEROMETER

Let us now consider the Mach–Zehnder interferometer (Fig. 8), which we will use in Sections 14 and 19 to analyze "intermediate phenomena." A beam of light is divided into two components at a beam splitter S_1, which we assume to have the same transmission and reflection coefficients $T^2 = R^2 = 1/2$ (in other words, half of the beam is transmitted, half is reflected). Each component is then reflected by a mirror and falls on another beam splitter S_2. Assuming for simplicity that the paths followed by each component (which we will call A and B) are exactly equal and that the beam splitters and mirrors are perfectly aligned, what will happen after passage through S_2? Within classical wave mechanics, we may think of the beam as a continuous wave, and consider[12] that a lossless symmetric beam splitter always introduces a phase lag between the reflected and the transmitted components of a quarter of a cycle, i.e., $\varphi_{\text{Refl.}} - \varphi_{\text{Trans.}} = \pi/2$. It is then straightforward to check that the two components that head to detector D_2 interfere destructively, while those that head to D_1 interfere constructively. All of the incident beam, therefore, will be detected at D_1 and nothing will fall on D_2.

The quantum regime may be attained by lowering the intensity of the beam so that only a few quanta enter the interferometer at a time, and by replacing the potentiometer detectors by photomultipliers or by equivalent solid-state devices that can single out individual photons. In this regime, only D_1 will register counts, nothing being registered at D_2 (except occasional background noise).

The phenomenon depicted in Fig. 8 is clearly undulatory. An interference pattern may be obtained by inserting a phase shifter H in path A, and slowly varying this phase shift

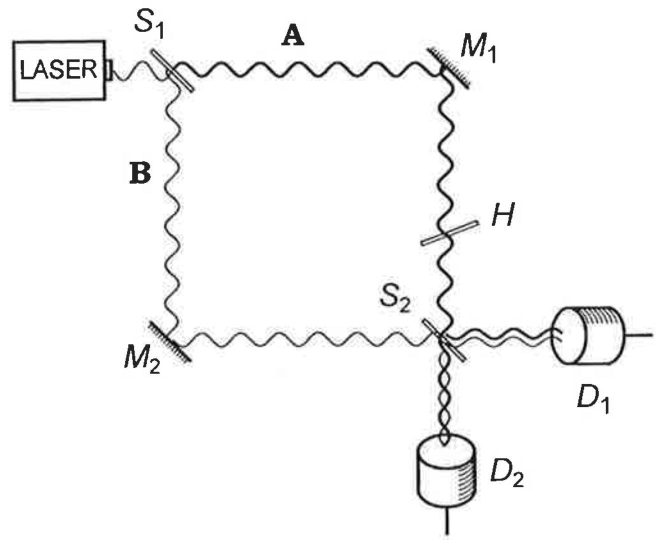

Figure 8. The Mach–Zehnder interferometer.

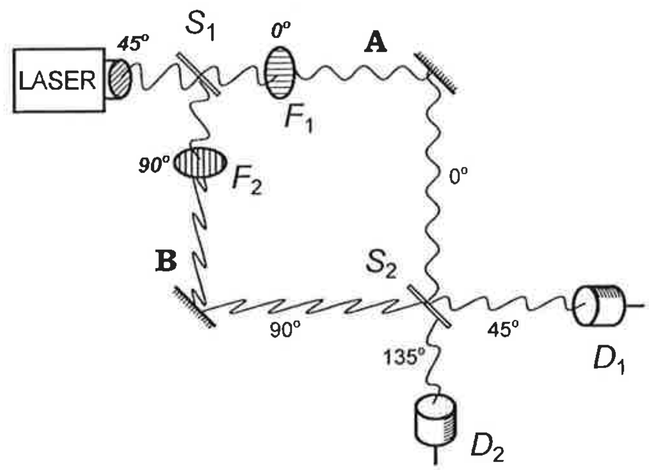

Figure 9. The Mach–Zehnder interferometer with a pair of orthogonal polarizers.

ϕ in time. One obtains a typical $\cos^2\phi$ variation of the intensity in both detectors (strictly speaking, of course, if there is a $\cos^2\phi$ variation in one detector, there will be a $\sin^2\phi$ variation in the other).

If one removes beam splitter S_2, one may infer the path of the photon after it is detected. If it falls on D_1, the path followed is B, if on D_2, then the retrodicted path is A. The phenomenon now is clearly corpuscular. The choice of removing or inserting S_2 may be delayed to a moment after the photon has passed through S_1.

Let us now introduce polarizers into the interferometer[13] (Fig. 9). Consider that the beam of light arrives at S_1 already linearly polarized along 45° (to improve the visibility of interference effects). A 0° polarizer is inserted in path A, and a 90° one in path B. What kind of phenomenon do we now have?

Reasoning within classical wave mechanics (which is adequate in this experiment, since each photon is not correlated with others), it is easy to see that waves oscillating in orthogonal directions cannot interfere destructively or constructively (in the sense of increasing the amplitude of oscillation along one of these directions). No interference occurs. In fact, if a phase shifter is introduced anywhere in path A and the phase shift ϕ is slowly varied with time, no variation of intensity will be measured at the detectors. The lack of interference suggests, according to the principle of complementarity, that the phenomenon should be corpuscular. But in the quantum regime, if a photon is detected at D_1, can we infer what path was followed? No. So in what sense is the phenomenon corpuscular? The explanation usually given is that "in principle" one could determine the path because the photon carries "which-path" information by means of its state of polarization. Let us refine this statement in the following section.

11. MODIFICATIONS THAT PRESERVE THE TYPE OF PHENOMENON

In Section 3, the complementarity of experimental arrangements was stated as the impossibility of having both interference patterns and retrodicted trajectories. This characterization allows for phenomena that don't exhibit interference and for which one cannot infer the trajectories of detected quanta. Still, these phenomena are readily classified into different *types*, i.e., as corpuscular or undulatory, according to what the scientist "in principle" could do. But what kinds of modifications can the scientist make in the apparatus that *do not* change the type of phenomenon?

We will suggest four classes of modifications that preserve the type of phenomenon. Others might have to be added to the list.

(i) *Replacement of a detector by an analyzer followed by an array of detectors.* Suppose that detector D_1 of Fig. 9 is replaced by a polarization analyzer (a bi-refringent prism) P_1 and two detectors D_1 and D_3, and that an analogous replacement is made for the other detector (see Fig. 10). (Such analyzers spatially separate an incident beam into two components, one having polarization along 0° and the other along 90°; each of the inserted detectors is placed to measure each of the components.) Then clearly the phenomenon is corpuscular. If a photon falls on detector D_3 of Fig. 10, one may infer that it followed path B, because its measured polarization is 90°. That is what Bohr's orthodox interpretation stipulates (not all interpretations will agree with it), and no inconsistencies arise from this stipulation.

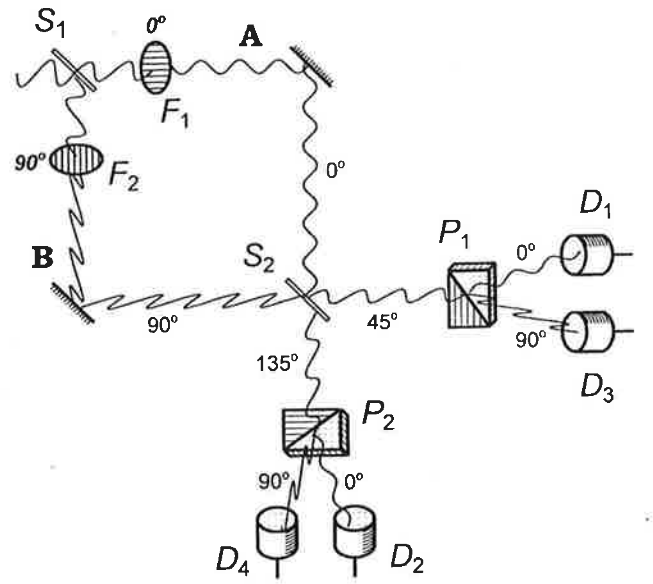

Figure 10. Modification of the Mach–Zehnder interferometer with polarizers that preserves the type of phenomenon.

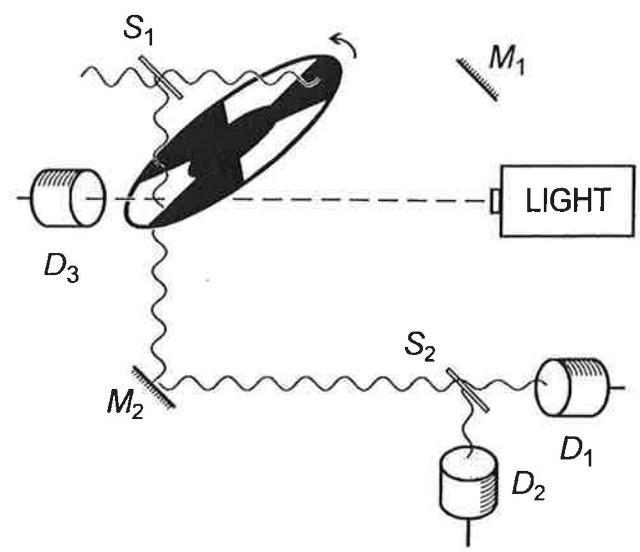

Figure 11. The Mach–Zehnder interferometer with a chopper.

Our proposal is that the transformation from the setup in Fig. 9 to that in Fig. 10 does not modify the type of phenomenon. Since the latter is clearly corpuscular, then the former also is, according to the proposed rule.

(ii) *Modification of a piece of apparatus that only interacts with other particles.* Special relativity applied to quantum mechanics imposes that nothing that is done in a faraway location can instantly affect the *statistics* of experimental outcomes in a laboratory. We will thus adopt the rule that a modification of a piece of apparatus at a distance cannot affect the type of phenomenon observed in a laboratory. In Section 6, this rule was used together with complementarity to show that if the particles used in a double-slit experiment are correlated with another system, no interference pattern will appear for these particles, even if the faraway system is never measured. If the faraway system was measured, then one would have information about the path of each particle in the laboratory, so that complementarity would prohibit interference. Since, according to the rule proposed here, no modification at a distance can affect the type of phenomenon in the lab, then the phenomenon is corpuscular even if no actual information is obtained about the path of the particles.

This macroscopic locality rule may be strengthened to any situation involving pairs of correlated particles. One could simply state that no modification of the apparatus that affects only the faraway system (be it space-like separated or not) can change the type of phenomenon in the lab.

(iii) *Simultaneous measurement of a "classical" system.* Consider a version of the Mach–Zehnder interferometer in which a mechanical chopper (a rapidly rotating toothed wheel) is inserted after the first beam splitter (Fig. 11). When the chopper lets through beam *A*, it blocks beam *B*, and vice versa. Is this a particle or a wave phenomenon?

Intuitively it is easy to see that no interference will occur since whenever one component arrives at S_2, the other is absent. But if a photon arrives at one of the detectors, the observer does not know which path was taken. Still, the phenomenon is corpuscular. One way to justify this is to consider that a simultaneous measurement of the angular position of the chopper may be performed without disturbing the quantum system. For example, a beam of light falling on a potentiometer D_3 may be used: the periodic blocking of this beam by the chopper is directly registered as a periodic absence of signal in the potentiometer. With this information, it is easy for the scientist to infer the path taken by a particular photon. We say that the measurement is performed on a "classical" object because it does not affect the outcome of the quantum measurement, in the interferometer. Such a characterization is basically circular, but let us not delve into the difficult question of defining the borderline between quantum and classical (it might help to think of a classical object simplistically as a quantum system in a coherent state, as presented in Section 9).

The phenomenon is the same, whether the measurement on the classical chopper is performed or not (according to this third proposed rule). Since one may infer trajectories by performing the measurement on the classical object, then the phenomenon without such measurement is also corpuscular.

(iv) *Insertion or variation of a phase shifter.* In the discussion of Fig. 8, it was mentioned that a variation of the phase shift ϕ in one of the arms of the interferometer leads to typical $\cos^2\phi$ and $\sin^2\phi$ variations in the respective counting rates of the detectors. This is the signature of a wave phenomenon, but there is a problem here: strictly speaking, each setting of the phase shifter corresponds to a different phenomenon. This is a curious situation: to show that a certain phenomenon (say with $\phi = 0$) is undulatory, one must change the phenomenon. This strategy can only work if all the phenomena obtained by varying the phase shift are of the same type. This in fact seems to be the case: the variation of the phase of any component does not modify the type of phenomenon.

These four rules seem sufficient for strengthening the principle of complementarity of experimental setups presented in Section 3. We may now state it as follows:

C_3. A specific "phenomenon" either shows clear interference patterns (or shows them through a phenomenon-type preserving modification)—being wave-like—or exhibits unambiguous trajectories (or exhibits them after a phenomenon-type preserving modification)—being particle-like.

As previously indicated, this characterization must still be refined to account for intermediate phenomena.

12. THE NATURE OF THE APPARATUS IS IRRELEVANT IN JUDGING THE PHENOMENON

The Mach–Zehnder interferometer working in the quantum regime constitutes a wave phenomenon. The insertion of orthogonal polarizers in the arms of the interferometer renders the phenomenon corpuscular. What if now a pair of polarizers oriented at 45° are inserted in paths A and B, after the orthogonal ones (Fig. 12a)? Thinking in terms of classical wave physics, the amplitudes that pass through the polarizers will be oscillating in the same direction when they meet at S_2, and since their coherence is maintained, there will be constructive superposition of the amplitudes heading to D_1, and destructive superposition of those heading to D_2. The phenomenon is undulatory.

In fact, one could test this by inserting a phase shifter in path A, as previously indicated. The variation of ϕ would lead to a $\cos^2\phi$ variation of the counting rate at a detector (and to a $\sin^2\phi$ variation at the other). An alternative test may be readily realized in a simple didactic Mach–Zehnder interferometer, in which the beams are slightly divergent and the mirrors imperfectly aligned. If the phenomenon is undulatory, students can observe an interference pattern if screens are substituted for the detectors. For corpuscular phenomena, the pattern disappears.

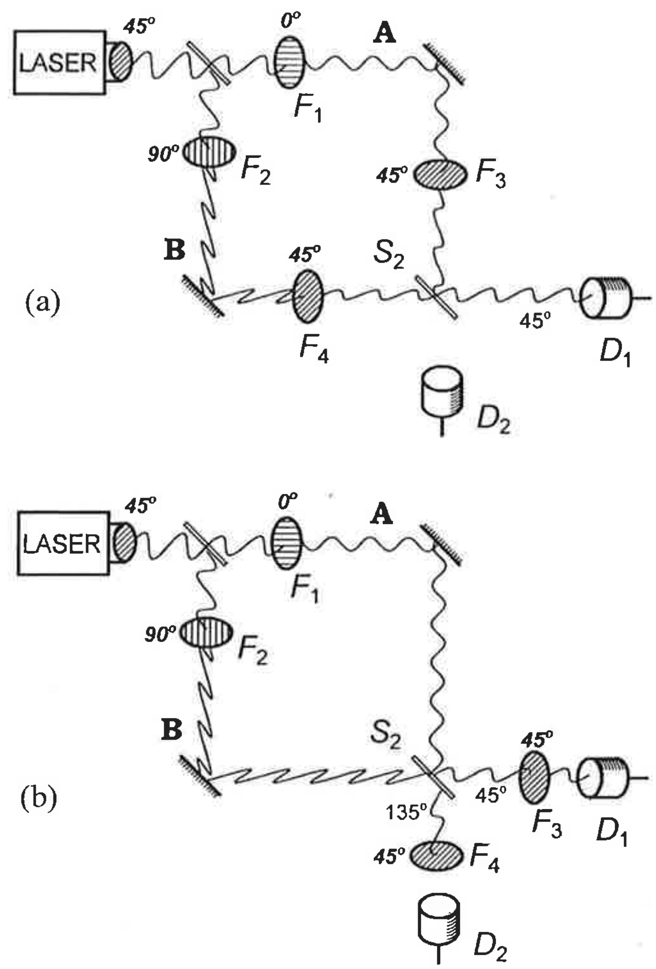

Figure 12. Insertion of polarizers at 45° in the Mach–Zehnder interferometer with orthogonal polarizers.

What if the polarizers are inserted *after* S_2 (Fig. 12b)? The situation right after S_2 is the same as that in Fig. 9. Reasoning according to classical wave mechanics, the component heading toward polarizer F_3 and detector D_1 turns out to have linear polarization at 45°, while the one heading toward F_4 and D_2 is polarized at 135°. When these amplitudes reach polarizers oriented at 45°, the first will be totally transmitted while the second will be completely blocked. What arrives at the detectors is the same as in the previous case of Fig. 12a. If a phase shifter inserted in one of the paths is varied, the typical $\cos^2\phi$ variation of the counting rate at each of the detectors is measured. The behavior of the system, as far as experimental outcomes are concerned—regardless of the nature of the components inserted in the apparatus—is undulatory.

This example illuminates a recent discussion in the literature on complementarity. Consider a photon "anti-correlation" experiment (Fig. 13a) in which a single photon passes through a beam splitter S_1 and falls either in detector D_1 or in D_2. The phenomenon is clearly corpuscular. Ghose et al.[14] consider a variation of this setup in which the beam splitter is replaced by a total internal reflection prism *RP* (Fig. 13b). Their argument is as follows. Since trajectories are known, the phenomenon should be corpuscular, but the quantum tunneling effect that takes place between the prisms and allows light to reach detector D_1 is essentially a wave effect, so that we would need *both* a particle and a wave picture to understand the experiment. This would be "irreconcilable with the usual formulation of the complementarity principle."[14]

The main problem with this argument is that it mixes a realist view of what is going on before detection ("tunneling is an essentially undulatory effect") with the more operationalist spirit involved in the principle of complementarity. The complementarity of experimental arrangements refers to detections of quanta and to the possibility of retrodicting trajectories; it refers to "phenomena" (in Bohr's sense), not to the nature or underlying mechanisms of the experimental components. I am not denying that the complementarity interpretation of Bohr might someday be consensually dropped infavor of a realist interpretation of quantum mechanics. The point is that Ghose et al.'s argument is irrelevant within the positivistic framework of complementarity. (Another argument may be given against their specific proposal: it is possible to elaborate a corpuscular view of tunneling, assuming that the energy of the particle can fluctuate to values above the barrier's potential, within the small time intervals allowed by the principle of uncertainty.[15]) Both phenomena in Fig. 13 are corpuscular.

If the nature of the apparatus's components is irrelevant for judging the type of phenomenon, then what is relevant? What is relevant is the temporal evolution of the quantum object (represented by its state) and the settings of the detectors that are actually triggered (represented by the "observable"). We show elsewhere[16] that it is possible to alter the type of phenomenon without in any way affecting the quantum-mechanical state.

13. "PHENOMENON" REFERS TO AN INDIVIDUAL QUANTUM

Consider now a slight modification of the setup of Fig. 12b. Suppose that polarizer F_4 was rotated from 45° to 0°. Now if a photon is detected at D_2, we can infer its trajectory (it came necessarily through path A), so it corresponds to a corpuscular phenomenon. If it is detected at D_1, the phenomenon is undulatory, as before (rule (ii) of Section 11 could be applied here). We therefore conclude that a phenomenon does not depend only on the nature of the quantum object and experimental setup, but also on where the individual quantum is detected.

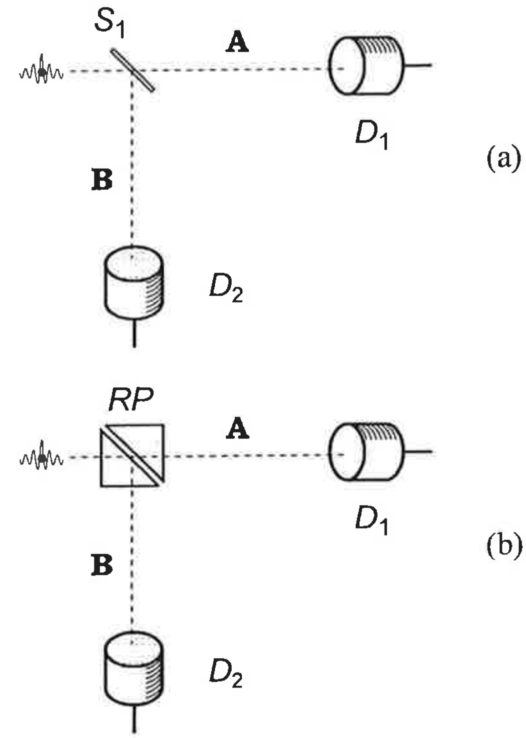

Figure 13. (a) Anticorrelation experiment; (b) Ghose et al.'s version with tunneling.

14. INTERMEDIATE PHENOMENA BETWEEN WAVE AND PARTICLE

What if now the polarizers F_3 and F_4 of Fig. 12b were rotated to intermediate angles between 0° (corresponding to a wave phenomenon) and 45° (corresponding to a corpuscular phenomenon), as in Fig. 14? We would have a truly *intermediate* phenomenon, between wave and particle! The interference pattern (obtained either by varying ϕ or for a slightly divergent beam with misaligned mirrors) would have a "visibility" between 0 (no pattern) and 1 (perfectly crisp pattern).

Such a possibility was only pointed out in the literature in 1979![17] It seems certain that Bohr never thought of it, in spite of its conceptual simplicity. At first sight, intermediate phenomena appear as a problem for the principle of complementarity, which states that wave and particle are mutually exclusive aspects of our description of the world. The existence of intermediate phenomena shows that these aspects may be superposed in some sense, so they are not really "mutually exclusive."

However, given an intermediary phenomenon, there is always a complementary phenomenon that is mutually exclusive to it. For example, the phenomenon in Fig. 14,

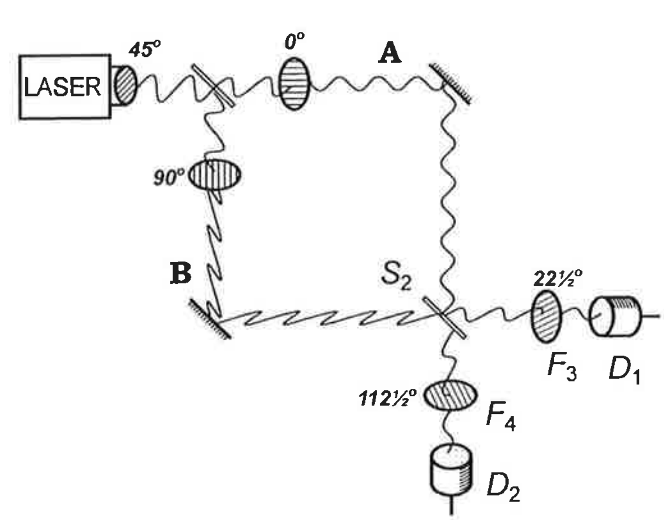

Figure 14. Intermediate phenomenon with polarizers.

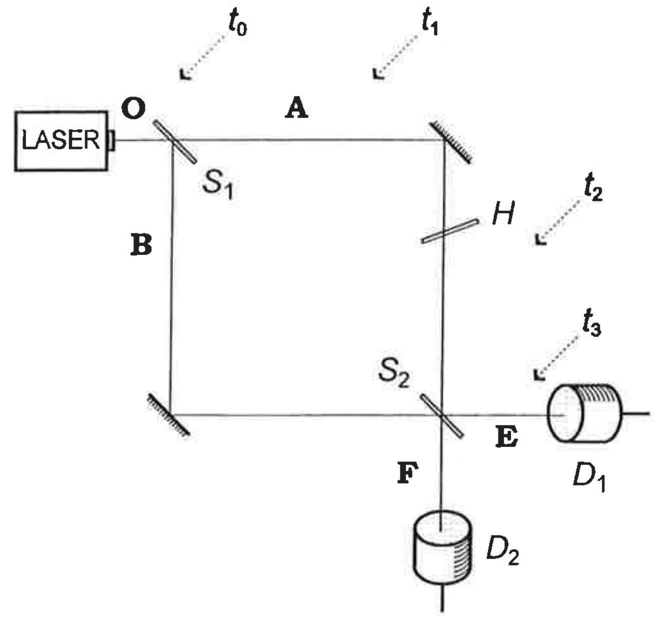

Figure 15. Mach–Zehnder interferometer with phase shifter and beam splitter S_2 with variable transmittance.

which may be characterized as being 50% corpuscular and 50% undulatory, is mutually exclusive to the one obtained by setting F_3 to 67.5° and F_4 to 157.5°. For other intermediary phenomena, obtained with other angles of polarization, how can one find the complementary phenomenon? What does it mean for two phenomena to be mutually exclusive? What is the connection between phenomena and observables? These questions will be addressed after a mathematical definition of retrodiction is given in the following section.

15. RETRODICTION TO PATHS

Let us now analyze a little more in depth the assumption of retrodiction and how it may be used to define complementary setups.

As our case study, we will consider a Mach–Zehnder interferometer in which one may vary the transmission and reflection amplitudes of beam splitter S_2, and also the relative phase ϕ between the component beams. The situation is depicted in Fig. 15. The incoming beam at time t_0 is represented by a generic state vector $|r_0\rangle$. After beam splitter S_1, at time t_1, the transmitted component is $2^{-1/2}|r_A\rangle$, which indicates that the position of the beam is along path A. The reflected component is $i(2^{-1/2})|r_B\rangle$, where the phase i is due to the quarter-wave advance of the reflected component in relation to the transmitted one, valid for lossless symmetric beam splitters.[12] The state at time t_1 is therefore

$$|\psi(t_1)\rangle = \frac{1}{\sqrt{2}}|r_A\rangle + \frac{i}{\sqrt{2}}|r_B\rangle. \tag{5}$$

Each component beam is then reflected at the mirrors (which introduce the same phase delays), while beam A also passes through a phase shifter H, which introduces a phase delay $e^{i\phi}$. The system then interacts with beam splitter S_2, which has a transmission amplitude given by T and a reflection coefficient R. These amplitudes are taken to be real numbers, with $T^2 + R^2 = 1$. The state at time t_3 right before detection becomes

$$|\psi(t_3)\rangle = \frac{i}{\sqrt{2}}(T + Re^{i\phi})|r_E\rangle + \frac{1}{\sqrt{2}}(Te^{i\phi} - R)|r_F\rangle, \tag{6}$$

where E is the path leading to detector D_1 and F to D_2.

The evolution from the state at time t_1 (5) to that at time t_3 (6) is described by a unitary evolution operator $\hat{U}_{T,\phi}(t_3, t_1)$, which depends on the values chosen for T, ϕ. The explicit expression for this $\hat{U}_{T,\phi}(t_3, t_1)$ is

$$\hat{U}_{T,\phi}(t_3,t_1) = Te^{i\phi}|r_F\rangle\langle r_A| + iR|r_F\rangle\langle r_B| + iRe^{i\phi}|r_E\rangle\langle r_A| + T|r_E\rangle\langle r_B|. \tag{7}$$

Its inverse may be readily calculated:

$$\hat{U}^{-1}_{T,\phi}(t_3,t_1) = Te^{-i\phi}|r_A\rangle\langle r_F| - iR|r_B\rangle\langle r_F| - iRe^{-i\phi}|r_A\rangle\langle r_E| + T|r_B\rangle\langle r_E|. \tag{8}$$

Let us now adopt a convention to simplify the notation concerning retrodiction. If a photon is for sure in path A, we may associate with this event the state $|r_A\rangle$. In what follows, we will refer to $|r_A\rangle$ as A, so that the superposition of (5) can be written as $2^{-1/2}A + i(2^{-1/2})B$.

First we will consider the corpuscular phenomenon, obtained by fixing $T = 1$ (and consequently $R = 0$), which is equivalent to the removal of beam splitter S_2 (for simplicity, also assume $\phi = 0$). If a detection occurs at D_1, what is the probability that the photon took path A?

Before answering this simple question, it is important to notice that we are implicitly assuming that at time t_0 the state of the system was $|r_0\rangle$, which we will abbreviate by O. The question we are asking, therefore, is what is the value of the *conditional probability* $\text{Prob}(A\,|\,O\&D_1)$, i.e., the probability that the state of the system at time t_1 was A given that the state at t_0 was O and that the photon was detected in D_1?

We know that if the photon is located for sure in path A, it will fall on D_2, and that if it is located in path B, it will fall on D_1. It is clear therefore that if the photon arrived at D_1, it must have come through B:

$$T=1,\quad \phi=0:\quad \begin{cases}\text{Prob}(A|O\&D_1)=0,\\ \text{Prob}(B|O\&D_1)=1.\end{cases} \tag{9}$$

Retrodiction may be defined as the calculation of the above set of conditional probabilities. In more general terms, given a specific experimental setup with initial state O, *retrodiction* is defined as the set of conditional probabilities $\{\text{Prob}(X_i\,|\,O\&D)\}$, for a certain detector D, referring to a set of states $\{X_i\}$ (which in the above example corresponds to a set of paths) at a certain time t.

Now consider the undulatory phenomenon examined in Section 10, in which $T = R = 2^{-1/2}$ and $\phi = 0$ (Fig. 8). In this case, if the wave-function associated with the photon was A at time t_1, it would have a probability 1/2 of arriving at D_1; if it was B, the probability of arrival in D_1 would also be 1/2. Since these probabilities are the same and there are no other alternatives at time t_1, we have

$$T=\frac{1}{\sqrt{2}},\quad \phi=0:\quad \begin{cases}\text{Prob}(A|O\&D_1)=\dfrac{1}{2},\\[2mm] \text{Prob}(B|O\&D_1)=\dfrac{1}{2}.\end{cases} \tag{10}$$

The "posterior probabilities" of (10) define a clear undulatory phenomena associated with detector D_1, referring totime t_1 and to the set of paths $\{A, B\}$. In other words, no "which-path" information is available in this type of phenomenon. In contrast to that, (9) corresponds to an unambiguous corpuscular phenomenon.

Consider now the case in which $T = R = 2^{-1/2}$ as above, but $\phi = \pi/2$. A brief inspection is enough to convince us that the counting rates in detectors D_1 and D_2 are the same, just as in the case of the corpuscular phenomenon. This phenomenon is also clearly undulatory, since it can be seen to satisfy the probabilities of (10).

So far, we have been considering retrodiction to the set of states $\{A, B\}$, where A and B are states with well-defined position, or paths. If retrodiction to paths furnishes the value 1 for one path (9), then the phenomenon is corpuscular. If retrodiction to paths furnishes the same probability for all paths, then the phenomenon is undulatory. In more general terms, for a certain detector D, initial state O, referring to a certain time t and to a set of paths $\{A_i\}$, where $i = 1$ to N, the phenomenon is *corpuscular* iff $\text{Prob}(A_i\,|\,O\&D) = \delta_{ik}$ for $1 \le k \le N$. The phenomenon is *undulatory* iff $\text{Prob}(A_i\,|\,O\&D) = 1/N$ for every i. Other cases constitute intermediary types of phenomena.

16. RETRODICTION WITH CERTAINTY

So far we have been able to define corpuscular and undulatory phenomena in terms of retrodiction to *paths*. But to account for the existence of complementary pairs of *intermediary* phenomena, one must consider retrodiction to any basis of states in the past. For example, consider the states $-i(2^{-1/2})A + 2^{-1/2}B$ and $2^{-1/2}A - i(2^{-1/2})B$, which span the same two-dimensional Hilbert space spanned by A and B. If we calculate the posterior probabilities for the three setups considered in the previous section, we obtain

$$T=1,\quad \phi=0:\quad \begin{cases}\text{Prob}\left(\dfrac{-i}{\sqrt{2}}A+\dfrac{1}{\sqrt{2}}B|O\&D_1\right)=\dfrac{1}{2},\\[2mm] \text{Prob}\left(\dfrac{1}{\sqrt{2}}A-\dfrac{i}{\sqrt{2}}B|O\&D_1\right)=\dfrac{1}{2};\end{cases} \tag{11}$$

$$T=\frac{1}{\sqrt{2}},\quad \phi=0:\quad \begin{cases}\text{Prob}\left(\dfrac{-i}{\sqrt{2}}A+\dfrac{1}{\sqrt{2}}B|O\&D_1\right)=1,\\[2mm] \text{Prob}\left(\dfrac{1}{\sqrt{2}}A-\dfrac{i}{\sqrt{2}}B|O\&D_1\right)=0.\end{cases} \tag{12}$$

For the setup with $T = 2^{-1/2}$ and $\phi = \pi/2$, the probabilities obtained are 1/2 and 1/2.

Equation (12) expresses an important concept, which we will call "retrodicted state with probability 1," or simply "retrodicted state with certainty." For the phenomenon with $T = 2^{-1/2}$ and $\phi = 0$, if a photon is detected in D_1, then the *retrodicted state with certainty* at time t_1 is $-i(2^{-1/2})A + 2^{-1/2}B$. What this means is that if the state at time t_1 is *orthogonal* to $-i(2^{-1/2})A + 2^{-1/2}B$, then there is probability 0 of detection at D_1. With this concept we have obtained a different criterion for characterizing the phenomenon as undulatory. The retrodicted state with certainty for this setup, referring to detector D_1, is a superposition *with equal weights* of the eigenstates A and B of well-defined position. This justifies calling this phenomenon undulatory.

Griffiths[18] has developed a general formalism for calculating retrodiction in his "consistent histories" approach to quantum mechanics. Simplifying his analysis, we have arrived at the following rules for finding the retrodicted state with certainty for a detector such as D_1.

(i) Apply the *inverse evolution operator* $\hat{U}_{T,\phi}^{-1}(t_3, t_1)$ (8) to the eigenstate $|r_B\rangle(t_3)$ associated with the detector D_1, obtaining a sum of amplitudes of the following form at time t_1: $\Sigma_i |r_i\rangle(t_1)$.

(ii) Multiply each amplitude $|r_i\rangle(t_1)$ in the sum by $|\langle r_0| \hat{U}_{T,\phi}^{-1}(t_1, t_0)|r_i\rangle$, which amounts to the application of the inverse evolution operator $\hat{U}_{T,\phi}^{-1}(t_1, t_0)$ to the initial time t_0 followed by taking the absolute value of the scalar product with the initial state O.

(iii) Normalize the resulting sum to obtain the retrodicted state with certainty associated with D_1.

For the setups associated with Fig. 15, steps 2 and 3 can be omitted, since they do not affect the final result. To illustrate this procedure, consider now the phenomenon with $T = 2^{-1/2}$ and $\phi = \pi/2$. For D_1, what is the retrodicted state with certainty? Applying $\hat{U}_{T,\phi}^{-1}(t_3, t_1)$ to $|r_B\rangle$, one obtains $2^{-1/2}A + 2^{-1/2}B$. For this phenomenon, we may write

$$T = \frac{1}{\sqrt{2}},\quad \phi = \frac{\pi}{2}:\quad \begin{cases} \text{Prob}\left(\frac{1}{\sqrt{2}}A + \frac{1}{\sqrt{2}}B \middle| O \& D_1\right) = 1, \\ \text{Prob}\left(\frac{i}{\sqrt{2}}A - \frac{i}{\sqrt{2}}B \middle| O \& D_1\right) = 0. \end{cases} \tag{13}$$

This is also an undulatory phenomenon since the associated retrodicted state with certainty is a superposition with equal weights of the eigenstates A and B.

The importance of retrodiction with certainty is dependent on the interpretation being adopted. We have seen, according to (9), that one is justified in saying that a quantum detected at F had a well-defined trajectory along path A (and one detected at E came along path B). But, of course, one may adopt a wave interpretation and still associate the superposition of the states in (5) as the best description at time t_1; retrodiction with certainty would therefore be only a formal expedient. For the complementarity interpretation, however, as well as for most ensemble interpretations, retrodiction with certainty is more than a formal trick: it carries ontological weight.

17. DEFINITIONS OF PHENOMENON AND COMPLEMENTARITY

We have arrived at satisfactory definitions of corpuscular and undulatory types of phenomena, but what does it mean for two phenomena to be *complementary*? It is sometimes stated that complementary phenomena correspond to incompatible observables. In this case, to what observables do the wave and particle phenomena in the Mach–Zehnder interferometer correspond? It is not the observables being directly measured at the detectors. The observables we are looking for will be those obtained, after the quanta are detected, by "retrodiction with certainty" to a certain instant in the past.

We have up to now examined three different phenomena. The corresponding retrodictions with certainty are indicated in (9), (12), and (13). Each has furnished a different basis of vectors spanning the two-dimensional Hilbert space. The corpuscular phenomenon with $T = 1$ and $\phi = 0$ furnished the paths $\{A, B\}$. The wave phenomenon with $T = 2^{-1/2}$ and $\phi = 0$ furnished the basis $\{-i(2^{-1/2})A + 2^{-1/2}B,\ 2^{-1/2}A - i(2^{-1/2})B\}$, while that with $T = 2^{-1/2}$ and $\phi = \pi/2$ furnishes $\{2^{-1/2}A + 2^{-1/2}B,\ i(2^{-1/2})A - i(2^{-1/2})B\}$.

A *phenomenon* may be defined as *the orthonormal basis obtained by retrodiction with certainty from the directly measured eigenstate.* Instead of defining phenomenon in terms of an orthonormal *basis*, one could use this basis to define an operator (by stipulating the eigenvalues associated with each basis vector), and then define phenomenon in terms of this operator (or the corresponding observable). But since the choice of eigenvalues is not always natural (especially in undulatory phenomena), we will not refer to an observable when characterizing a phenomenon, but to an orthonormal basis.

Complementary phenomena[19] may be formally defined as those *associated with "mutually unbiased" bases.* An example of three mutually unbiased bases[20] are the bases of eigenstates associated with the spin-component observables $\hat{\sigma}_x$, $\hat{\sigma}_y$, and $\hat{\sigma}_z$. If a state is prepared as an eigenstate of $\hat{\sigma}_x$, for example, then a measurement of $\hat{\sigma}_y$ has an equal probability of yielding the different possible outcomes. One may restate this by saying that a state prepared as a projector onto one of the vectors composing a basis "gives no information" about what the outcome will be for a measurement of the observable whose eigenstates form a basis that is mutually unbiased to the former one.

Two bases $\{\psi_{11}, \psi_{12}, ..., \psi_{1N}\}$, $\{\psi_{21}, \psi_{22}, ..., \psi_{2N}\}$ in a Hilbert space of finite dimension N are *mutually unbiased* if and only if, for every pair ψ_{1i}, ψ_{1j},

$$|\langle \psi_{1i}|\psi_{2j}\rangle|^2 = \frac{1}{N}. \tag{14}$$

For a space of dimension N, the maximum number[20] of mutually unbiased bases is $N + 1$.

18. DOES COMPLEMENTARITY ONLY INVOLVE PAIRS?

The Hilbert space associated with the eigenstates $|r_A\rangle$ and $|r_B\rangle$ has dimension $N = 2$, so one expects three mutually unbiased bases, just as in the case of the spin-1/2 observables mentioned above.

In the case study we have been examining, we have already arrived at three mutually unbiased bases:

- $T = 1$ and $\phi = 0$: basis $\{A, B\}$, corpuscular phenomenon;
- $T = 2^{-1/2}$ and $\phi = 0$: basis $\{-i(2^{-1/2})A + 2^{-1/2}B, 2^{-1/2}A - i(2^{-1/2})B\}$, "primary" wave phenomenon; and
- $T = 2^{-1/2}$ and $\phi = \pi/2$: basis $\{2^{-1/2}A + 2^{-1/2}B, i(2^{-1/2})A - i(2^{-1/2})B\}$, "secondary" wave phenomenon.

We have arrived at a conceptual problem! Are the two different wave phenomena complementary to each other? According to the formal definition presented above, yes. But, of course, Bohr's idea of complementarity was different. For him, a *pair* of phenomena, embodying the wave and particle aspects, are not only mutually exclusive but they also *exhaust* all aspects of the quantum phenomenon (they are mutually complete): "the impossibility of combining phenomena observed under different experimental arrangements into a single classical picture implies that such apparently contradictory phenomena must be regarded as complementary in the sense that, taken together, they exhaust all well-defined knowledge about the atomic objects."[21]

As we have seen, however, the corpuscular and the primary wave phenomena do not exhaust all the knowledge about the quantum object. To determine the pure state of an ensemble of quantum objects defined in a two-dimensional Hilbert space, one would need to measure three mutually unbiased observables, not just two. The idea that complementarity should not only apply to pairs of phenomena, but to $N + 1$ phenomena, was implicit in von Neumann's remark, upon learning about Bohr's principle of complementarity, that "there are many things which do not commute and you can easily find three operators which do not commute."[22]

Thus, pairs of phenomena (as defined above) do not exhaust all aspects of the quantum object. Complementarity, as defined above, *does not* only involve pairs of formally defined phenomena, but $N + 1$ of them.

There is, however, another way of defining an undulatory phenomenon. Consider the example given in Section 11 (item (iv)), in which a wave phenomenon is operationally characterized by varying a phase shift ϕ and observing a $\cos^2\phi$ pattern in the counting rate of a detector. To justify this procedure, it was suggested that this operation does not vary the kind of phenomenon. So according to an *exhaustive version* of complementarity, a wave phenomenon is any one whose basis (obtained by retrodiction with certainty from the directly measured eigenstate) is mutually unbiased with the basis of the corpuscular phenomenon. In this case, we might say that complementarity involves the pair of *aspects* "particle" and "wave."

Summarizing this distinction, we find that we have two possible definitions of phenomena: (I) The *formal definition of phenomena* implicitly adopts the idea of *symmetry of representations* in quantum mechanics. According to this idea, any representation (in coordinate space, in momentum space, etc.) is equivalent, and none of them has a special ontological status (in spite of the fact that all direct measurements involve determinations of positions, not momenta). In this case, complementarity does not only involve pairs. This approach, however, has difficulty in giving an operational rule for characterizing the different kinds of wave phenomena. (II) The *exhaustive definition of phenomena* does not have this difficulty, but it is implicitly nonsymmetric, assuming that the coordinate representation has a special ontological status over the others. In this case, we may say that complementarity involves pairs of "aspects." The wave aspect would thus include different undulatory phenomena (obtained by varying ϕ).

Intermediate phenomena pose a serious problem to the exhaustive definition of complementarity. Such phenomena are characterized by a basis, obtained by retrodiction with certainty, that is not mutually unbiased with respect to the set of paths of the corpuscular phenomenon. For the Mach–Zehnder setup we have been examining, intermediary phenomena come in triplets: which one is to be taken as privileged in order to ensure the exhaustiveness of pairs?

19. INTERMEDIATE PHENOMENA IN THE MACH–ZEHNDER INTERFEROMETER

The Mach–Zehnder interferometer with variable phase shift ϕ and variable transmission amplitude T for beam splitter S_2 (Fig. 15) is a suitable system for exploring intermediate phenomena.[23] For values of T^2 other than 0, 1/2, and 1, the phenomenon is intermediary between particle and wave, as indicated in Fig. 16, where purely corpuscular and undulatory are also indicated.

Given any phenomenon, one may find phenomena complementary to it by writing out the bases obtained by retrodiction with certainty from the directly measured eigenstates, and checking if they form a mutually unbiased set of bases. For instance, restricting ourselves to $\phi = 0$, what would be the phenomenon complementary to that with $T_0^2 = 0.2$? A rough guess might indicate that it is one with $T_1^2 = 0.8$, but that is incorrect. Using (14), one finds that the phenomenon complementary to that with $T_0^2 = 0.2$ is one with $T_1^2 = 0.9$. The third phenomenon complementary to these two, according to the formal definition of complementarity, is the secondary wave phenomenon that we have already considered, with $T = 2^{-1/2}$ and $\phi = \pi/2$. Fig. 17 presents a plot of complementary pairs for every value of T_0^2, where T_0, T_1 refer to the transmission amplitudes of pairs of complementary phenomena.

Returning to Fig. 16, we have indicated the "primary" and the "secondary" undulatory phenomena, within the spirit of the formal definition of phenomena. It is easy to plot the wave aspect (within the exhaustive definition) by including all values of ϕ for $T^2 = 1/2$. One might now ask whether the phenomenon-type preserving modification (iv) of Section 11 is still valid for intermediate phenomena. In this example it is.

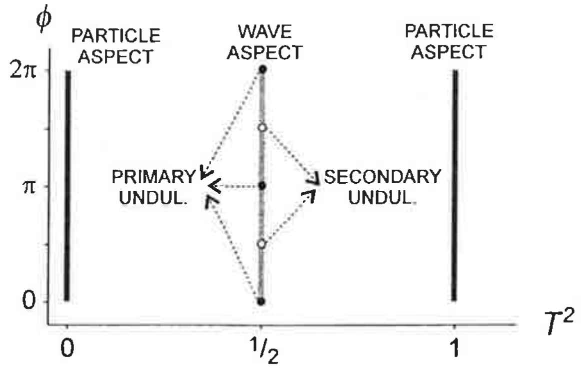

Figure 16. The space of phenomena spanned by ϕ and T^2. Corpuscular, primary, and secondary wave phenomena are indicated, as well as the set comprising the purely wave aspect.

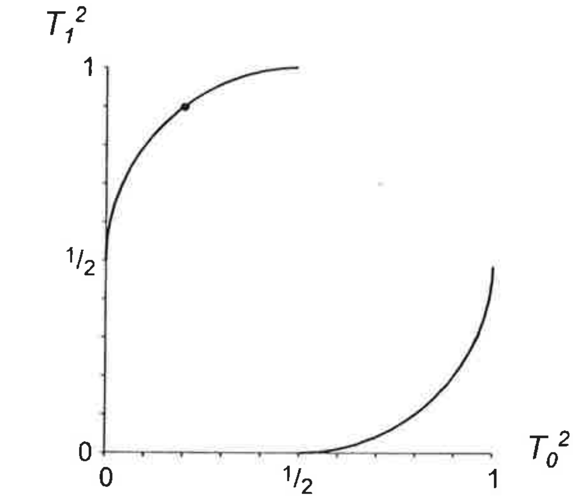

Figure 17. Set of complementary pairs of intermediate phenomena for $\phi = 0$.

To see this, consider that what characterizes an intermediate aspect is the value of the scalar product appearing in (14), where one basis corresponds to the corpuscular phenomenon and the other is obtained by retrodiction with certainty from the directly measured eigenstates:

$$|\langle r_A|\hat{U}^{-1}_{T,\phi}(t_3,t_1)\,|r_E\rangle|^2 = T^2.$$

Since the value obtained is always independent of ϕ, one may apply rule (iv) of Section 11 for intermediary phenomena also.

20. CONCLUSIONS

This article has explored the principle of complementarity first proposed by Bohr, showing in what sense it is a useful synthesis of interesting aspects of the quantum-mechanical formalism. If an interference pattern of maximum visibility is observed by varying the phase shift of some component, the phenomenon is undulatory. If a trajectory can be inferred by retrodiction, it is corpuscular. The usual statement of the principle prohibits the coexistence of wave and particle phenomena, which amounts to a prohibition of observing interference and retrodicting trajectories. This principle has explanatory power, helping us to qualitatively predict what will happen in different interferometric experiments.

Two attempts to explain the principle of complementarity have been presented: either invoking a disturbance of the apparatus on the quantum object, or formally, by means of the orthogonality of the apparatus states. The problem of relating the two has been left open.

The nature of the components of the apparatus is irrelevant in judging the "type" of phenomenon. This type depends on what the initial quantum state is, how the macroscopic apparatus modifies the state, and which detector is triggered. For the same setup, the detection of different quanta may correspond to different types of phenomena. It also is possible to alter the type of phenomenon without in any way affecting the state of the quantum system, as we show elsewhere.[16]

Retrodiction with certainty from the directly measured eigenstates plays an essential role in the definition of a phenomenon. According to the formal definition, complementary phenomena are those with mutually unbiased bases, after performing retrodiction with certainty. In this case, the symmetry of representations in quantum mechanics is maintained, and complementarity should refer to sets of $N + 1$ phenomena, not only two. The extension of the principle of complementarity to intermediate phenomena is unproblematic if one adopts this formal definition.

The exhaustive definition of complementarity is closer to Bohr's original ideas, allowing one to sustain the descriptive completeness of pairs of phenomena. This is done by privileging corpuscular phenomena, and grouping all other phenomena (which are mutually unbiased in relation to well-defined paths) as the "wave aspect." Intermediate phenomena pose a serious problem to this definition.

In order to apply the principle of complementarity to every experiment in quantum physics, one may define modifications that preserve the type of phenomenon. In other words, if a type-preserving modification leads to the possibility of retrodiction with certainty to trajectories, then the original phenomenon is also corpuscular. Four such modifications were proposed, the latter of which, variation of phase shift, preserves the wave or particle "aspects," although it does alter the type of phenomenon as formally defined.

Let us now give a final version of the principle of complementarity that includes intermediary phenomena and favors the formal definition of the principle.

C_4. A specific "phenomenon" may show clear (visibility 1) interference patterns (or show them through a phenomenon-type preserving modification)—being wave-like—or may exhibit by retrodiction unambiguous trajectories (or exhibit them after a phenomenon-type preserving modification)—being particle-like. A phenomenon, defined by the setup of the apparatus and by a particular detection of a quantum, cannot be both wave-like and particle-like, although it may be intermediary, exhibiting an interference pattern with visibility between 0 and 1. In general, complementary phenomena may be formally defined as those associated with mutually unbiased bases obtained by retrodiction with certainty. Intermediary phenomena are characterized by having retrodicted bases that are not mutually unbiased with respect to those of corpuscular phenomena. This formal definition contradicts the thesis that complementary pairs of phenomena "exhaust" all information concerning a quantum-mechanical object.

APPENDIX: BOHR'S THREE TYPES OF COMPLEMENTARITY

Here we present a philosophical study of the three types of complementarity in quantum physics that appear in the writings of the Copenhagen group. In the paper, we have emphasized the complementarity of experimental arrangements, which asserts that an experimental setup that constitutes a wave phenomenon cannot at the same time be a corpuscular one. Such setups are "mutually exclusive," although both descriptions are supposed to be "exhaustive."

This mutual exclusion is analogous to the empirical claim that it is impossible to measure simultaneously (with as good a resolution as one could want) the position and the momentum of a particle. Some authors[24] have proposed experimental setups that could allegedly measure position and momentum with a resolution that would violate the uncertainty principle, but these proposals either don't work or depend on the acceptance of retrodiction (as discussed in Section 4 in relation to Fig. 3b).

So there is an analogy between the wave-particle pair and the position-momentum pair. The wave-particle aspects as well as the position-momentum observables involve a pair that cannot be simultaneously observed in a sharp way. There is, however, a big difference between these pairs: wave and particle are *mutually exclusive* in classical physics (you can't have both at the same time because each is the logical negation of the other: a wave is spread out while a particle is localized; a wave may be continually divided, while a particle is indivisible, within a certain energy domain); on the other hand, position and momentum are always *jointly well defined* in the classical mechanics of particles.

This difference indicates that there are different types of complementarity, which may be grouped into three, in agreement with an analysis by von Weizsäcker.[25]

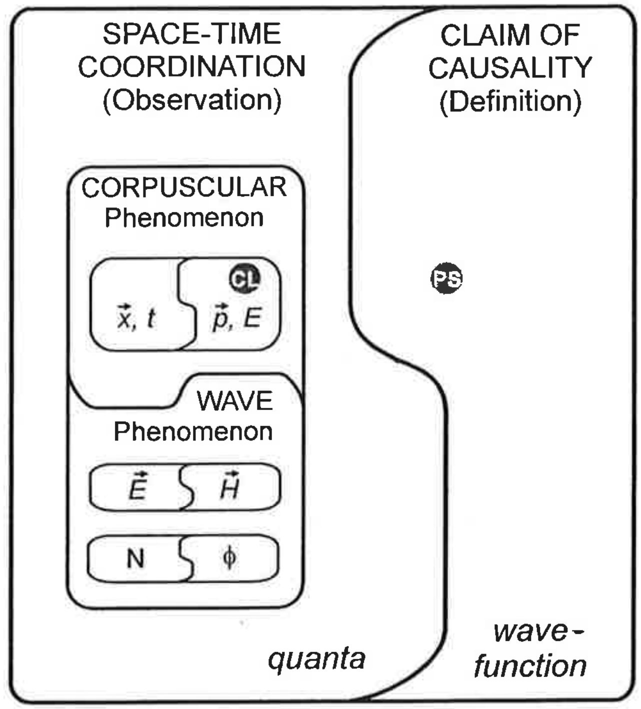

Figure 18. Diagram representing the three types of complementarity.

(i) *Complementarity between space-time coordination and the claim of causality (CSC).* In 1927, Bohr introduced the principle of complementarity by means of a beautiful (although sometimes ambiguous) argument, which he would however later abandon. It involved the mutual exclusion, in quantum physics, between the "space-time coordination" that arises from measurement (involving the quantum postulate), with the implication that the object system is open, and the "claim of causality" associated with the deterministic evolution of quantum states that may be ascribed to closed quantum systems (described by the Schrödinger equation). Bohr abbreviated these terms respectively as "observation" and "definition." This form of complementarity involves aspects that are *consistent* in classical physics. In Fig. 18, this type is represented by the outermost division (a rectangular yin-yang symbol) of the figure. After 1928, however, Bohr abandoned this type of complementarity, because it ran against the positivistic climate of those times:[26] how could one meaningfully distinguish an observed atom from an unobserved atom? Nowadays this type of complementarity should be seriously reconsidered by realist philosophers of physics.

(ii) *Complementarity between wave and particle (CWP).* Within the realm of observation (as opposed to definition) spelled out by the CSC, one has either a corpuscular or a wave phenomenon (see the intermediary yin-yang symbol drawn with a dashed line in Fig. 18). Bohr developed this conception between 1929 and 1935, and it involves aspects that are *mutually exclusive* in classical physics (contrary to the other two types). It is this complementarity of experimental setups that is being explored in this paper.

(iii) *Complementarity between incompatible observables (CIO).* This type was especially emphasized by Pauli,[27] being synonymous with the uncertainty principle. The complementarity between position and momentum or between time and energy (see small yin-yang symbol with the labels $\vec{x}$, t and $\vec{p}$, E in Fig. 18) involves aspects that are *consistent* in the classical mechanics of *particles*. What would be analogous complementary pairs within classical *wave* mechanics? Heisenberg[28] suggested a complementarity between electric and magnetic fields $\vec{E}$, $\vec{H}$. One could speculate that the well-known uncertainty relation in quantum field theory between particle number N and phase ϕ could also be understood as a CIO between two aspects that are consistent in classical wave mechanics: wave intensity (the square of the amplitude) and phase.

In his first paper, Bohr[29] also referred to the complementarity between the principle of superposition (wave propagating in space and time) and conservation laws (a photon conserves momentum and energy), which seems to involve a mixture of the different types of complementarity. When speaking of the principle of superposition (PS in Fig. 18), in the mentioned passage, Bohr seems to be thinking of the "claim of causality" of the CSC (and not of the wave phenomenon of the CWP); referring to conservation laws (CL in Fig. 18), this seems to apply to the momentum (or energy) observables within the CIO (which unfolds the corpuscular picture of the CWP).

As an illustration of the second and third types of complementarity, consider the Compton effect, which in 1923 was explained in terms of the conservation laws of classical mechanics of particles. It is a *corpuscular* phenomenon according to the CWP, since after the measurement of the scattered pair of particles (electron and γ-ray photon), one may infer (by retrodiction) the trajectory taken by each one. But does that mean that the exact position of the collision could be known in advance? No, that would introduce an uncertainty in the momenta that would invalidate the laws of conservation. Thus, with regard to the CIO, the Compton effect was classed by Bohr as involving well-defined momenta, not positions, in spite of being a corpuscular phenomenon.[30]

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Résumé

Cet article étudie divers aspects du principe de complémentarité de la physique quantique et on suggère certaines propositions pour le raffiner et l'étendre. Après une introduction à la dualité onde-corpuscule et aux concepts de Bohr sur la complémentarité, nous analysons l'expérience de la double fente pour une paire de corpuscules corrélées et examinons les propositions sur l'explication ultime de la complémentarité. Des propositions pour raffiner le principe sont exposées lorsque nous explorons des variations de l'interféromètre de Mach–Zehnder. On présente quatre classes de "modifications préservant le type du phénomène", ce qui justifie d'appeler le phénomène "corpusculaire" même quand les trajectoires ne peuvent pas être inférées. Nous soulignons que le terme "phénomène", qui se réfère à l'état de l'objet et à la disposition de l'appareil, doit aussi référer au quantum particulier qui est détecté, alors que la nature spécifique de l'appareil n'est pas importante pour juger un phénomène. La dernière partie de l'article présente une définition formelle du "phénomène" comme une base orthonormée obtenue par "rétrodiction avec certitude" ainsi que de "phénomènes complémentaires" comme ceux associés aux "bases mutuellement non biaisées. Ensuite nous posons la question de savoir si la complémentarité comprend seulement des paires, et examinons deux solutions alternatives. La définition formelle affirme qu'il y a N + 1 phénomènes mutuellement complémentaires (pour un espace N-dimensionnel), tandis que la définition complète associe le même "aspect ondulatoire" à tous les phénomènes qui sont mutuellement non biaisés par rapport à l'hypothèse corpusculaire. Les phénomènes intermédiaires sont analysés brièvement dans le cas d'un diviseur de faisceau avec transmissibilité variable et trouvés problématiques pour la définition complète de la complémentarité. Nous concluons ainsi, en présentant une définition de complémentarité qui soutient une approche formelle.

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