Reversibility and the Interpretation of Mixtures in Quantum Mechanics[1]

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The term "experimental philosophy" has been used to refer to the solution of what were considered philosophical problems by means of laboratory experiments. A recent example of this was the experimental violation of the Bell inequality, which ruled out certain philosophically appealing "realist local" theories as alternatives to quantum mechanics (QM).

Following the spirit of such experimental philosophy, this paper proposes a feasible test between two different interpretations concerning the nature of "mixtures" in QM. The use of delayed coincidence techniques seems to show that the process of mixing beams of light in different polarizations is reversible, favoring a weak version of the so-called "ignorance interpretation" over the "instrumentalist" view.

The first two sections survey in a conceptual way the philosophical discussion about the interpretation of mixtures. This is followed by a review of the mathematical notation and of procedures for preparing and analyzing mixtures. The argument that differently prepared but equivalent mixtures may be distinguished by measuring particle fluctuations is then showed not to be valid, at least in the example considered. This leads the way for the experimental argument proposed in favor of the ignorance interpretation.

1. The problem of interpretation of mixtures

"Given a beam of unpolarized electrons, should one think of each electron as having a definite spin orientation?" With these words, U. Fano (1957, p. 74) posed the problem of the *interpretation of mixtures* in QM. A beam with definite spin polarization corresponds to a "pure state" in QM, being represented by a state vector, while a beam of unpolarized electrons is usually associated with a "mixed state", being represented by a density operator. A single density operator can be resolved into many different combinations of pure states. Thus, the problem is whether the mixed state representing a given system should be thought of as a specific but unknown combination of pure states, or as a unique state in its own right.

In optics the discussion goes back to the 30's, when Birge had pointed out that there was "no scientific reason for the assumption, so commonly made in texts, that

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unpolarized light consists of *plane*-polarized components, oriented in all azimuths". He therefore concluded that "no experiment can give us any information on the nature of unpolarized light", besides the fact that "*if* such light is split into components, in any given apparatus, no preferential polarization will be found" (Birge, 1935, pp. 180, 182). If that is so, then the acceptance of an "instrumentalist" point of view would lead to the conclusion that *any* beam of unpolarized light is in the *same* state of polarization. That was the philosophical step taken by Fano (1957, p.76), for whom the definition of state should only be concerned with predictions of measurement outcomes in future experiments.

The opposing view, according to which an unpolarized beam of light consists of photons in definite but unknown polarization states, has been called the "ignorance" interpretation (IgI) of mixtures (term due to Putnam 1965, p. 98). It was the traditional view in statistical QM (see Fano, p. 76, and Park 1968, pp. 215-6), which separated clearly between the "objective" probability of measurement outcomes associated with pure quantum states, and the "subjective" probability arising from our incomplete knowledge of the microscopic state of a many-particle system.

Within this interpretation, one can speak of *different but equivalent* mixtures. This is the case of two beams A and B which are prepared by mixing different pure beams, but which yield the same mean values for any observable measured on the beams. If the preparation procedure is unknown, the ignorance view still maintains that equivalent mixed beams may be different.

In spite of being considered "unrealistic" by Fano (p. 74), the IgI can be classified as a "realist" view, since it conceives that distinct but indistinguishable physical states may underlie a same observable phenomenon. The belief in such reality arises from reasons other than empirical adequacy, reasons such as simplicity or uniformity of the physical theory. A mixture considered within the IgI will be referred to as a "realist" (or "classical") mixture. The "realism" of the IgI is however of a different sort from what is usually referred to as "the realist interpretation of quantum mechanics". For the latter, the elements of reality which underlie the observed phenomenon are variables which assign, for instance, well defined position and momentum to particles. The IgI, in contrast, is closer to what can be called "realism of the wave function" (the "naive realism" of Pearle 1986, p. 442), a view that assigns some sort of reality to probability amplitudes in configuration space.

2. The philosophical debate

In the 70's the discussion about the nature of mixtures became rather intense in the philosophical literature. We will examine an important argument in favor of the IgI in section 5, concentrating here on two different debates that took place, both involving *correlated* quantum mechanical systems. The typical example of such a composite system is the Einstein, Podolsky & Rosen (EPR) setup. In Bohm's (1951, pp. 614-619) version of the EPR setup, the correlated systems are two particles "entangled" with opposite spins, described by a pure composite state which has cylindrical symmetry about the trajectory of the particles. Another much explored example arises in the formal theory of measurements in QM. The measuring apparatus is considered a quantum system which becomes correlated to an object system during the measurement interaction.

The first debate arose after an argument due to van Fraassen (1972, pp. 325-31) that the IgI leads to inconsistencies. As pointed out by Hooker (1972, pp. 97-106) and Grossman (1974), implicit in van Fraassen's critique was the acceptance of the so-called "reduction assumption". This assumption applies to correlated systems

which are described by a pure composite state. If such a state is non-factorable (entangled), then one cannot assign a pure state to each individual subsystem. For measurements performed on only one of the subsystems, the best available description of the subsystem is considered to be a "reduced density operator" obtained by means of the mathematical operation of taking a "partial trace" (see for instance d'Espagnat 1976, pp. 58-61). The reduction assumption accepts such an "improper mixture" as the actual state of the subsystem. There are no compelling reasons to accept such an assumption, so that the IgI can be sustained. There have even been attempts to dissolve the inconsistencies without rejecting neither the IgI nor the reduction assumption, within the framework of quantum logic (Gibbins 1983).

The problem raised for the IgI by the study of correlated systems is not that of formal inconsistencies. The problem is that when describing the state of a beam, we usually do not know whether the particles of the beam are correlated to other unobserved systems. Thus, given a beam of unpolarized light, we might have a classical mixture of pure states, or we might have one subsystem of an entangled composite pure state. This possibility weakens the ignorance interpretation. Our initial characterization of this realist view, to be called the *strong IgI*, would have to be conditioned on the requirement that the beam is not correlated to other systems in the environment, which is quite a stringent condition. On the other hand, the *weak IgI*, which considers the possibility of correlations with the environment, does not answer Fano's question in the affirmative. A beam of unpolarized electrons might *not* consist of electrons in well-defined spin states (before a measurement takes place), if such electrons are correlated to other particles.

The second debate begun in the early 70's involved A. Fine's (1970) "insolubility proof" to the "measurement problem". Such a proof applies to *unitary* measurement interactions, *i.e.* it only involves the Schrödinger equation and not the projection postulate. Fine was able to show that there is no set of apparatus "pointer states" such that for any initial object state, the final composite state is in a realist mixture involving such pointer states. The important point for us here is that Fine's proof made use of the IgI, and assumed a special rule for the evolution of mixtures, which H. Brown (1986) called "real unitary evolution" (RUE). Such a rule applies the usual unitary evolution to each pure subsystem composing the mixture, leading to a new realist mixture. RUE prohibits replacing such an evolved state by any other mixture which according to the instrumentalist interpretation is equivalent to it.

The whole project of formulating insolubility proofs was criticized by Park (1973), who attacked the IgI which underlies Fine's approach. Contrary to Park's contention, however, the acceptance of his instrumentalist critique does not undermine the several *insolubility* proofs proposed in the literature, but only a possible *positive* solution to the measurement problem which would involve apparatus mixtures (see Pessoa 1990, p. 101).

Park (1968, pp. 214-7) is probably the most vocal defender of the instrumentalist position, within the framework of the "statistical interpretation of QM". According to this widespread view, the state vector does not refer to an individual system, but only to an "ensemble" of identically prepared systems. Park therefore does not have to address the additional problem of whether an unentangled individual particle can be in a impure mixture. According to the IgI, a single uncorrelated particle is always in a pure state, although our lack of knowledge might allow us to describe such a system as a mixture.

3. Mathematical notation

Having surveyed the problem of interpretation of mixtures in a conceptual way, let us introduce the mathematical notation for describing pure states and mixtures, and look at the experimental procedures for obtaining light beams in such states.

A pure state can be represented by a normalized vector $|\phi\rangle$ in an appropriate Hilbert space $\mathcal{H}$. According to the standard approach, the measurement of an observable represented by an operator Q in $\mathcal{H}$ yields as possible outcomes the eigenvalues a_i associated with the eigenvectors ϕ_i of Q. If the pure state is written as a superposition of such eigenvectors, $|\phi\rangle = \sum_i a_i \cdot |\phi_i\rangle$, then the probability for an outcome a_i is given by $|a_i|^2$.

A mixture is represented by a density operator $\hat{W} = \sum_j w_j \cdot \beta[\Psi_j]$ acting on $\mathcal{H}$. The coefficients w_j give the probability of obtaining the eigenvalue β_j as the outcome of the measurement of an observable $\hat{R}$ with eigenvectors $|\Psi_j\rangle$. The operator $\beta[\Psi_j]$ projects any state vector onto the one-dimensional subspace spanned by $|\Psi_j\rangle$, and can be written[2] as $\hat{P}[\Psi_j] = |\Psi_j\rangle\langle\Psi_j|$. A mixture for which $\hat{W}^2 = \hat{W}$ corresponds to a pure state.

Now let us suppose that we are going to measure the observable represented by $Q = \sum_i \alpha_i \cdot \hat{P}[\phi_i]$ for the system represented by the density operator $\hat{W} = \sum_j w_j \cdot \hat{P}[\Psi_j]$. What are the probabilities for obtaining the different eigenvalues α_i of Q? One way of calculating this is by transforming from the basis $\{\Psi_j\}$ to the basis $\{\phi_i\}$, using a set of equations $|\Psi_j\rangle = \sum_i c_{ji} \cdot |\phi_i\rangle$. We would obtain: $\hat{W} = \sum_i \sum_{i'} \cdot v_{ii'} \cdot\cdot |\phi_i\rangle\langle\phi_{i'}|$, where $v_{ii'} = \sum_d w_j c_{ji} c_{ji'}^*$. The matrix $[v_{ii'}]$ is the "density matrix" in the representation $\{\phi_i\}$. The diagonal elements v_{ii} furnish the probabilities for measuring the eigenvalue α_i, while the off-diagonal terms $(v_{ii'}\ (i \neq i')$ express the "coherence" of the state, the fact that the system cannot be represented as a classical mixture of pure states $|\phi_i\rangle$. The density matrix can be shown to be self-adjoint ($v_{ii'} = v_{i'i}^*$) and positive definite (v_{ii} is real and ≥ 0), with unit trace ($\sum_i v_{ii} = 1$) (Fano, 1957, p. 77). In the representation $\{\Psi_j\}$, the density matrix has diagonal elements w_j and null off-diagonal elements.

Any density matrix may be diagonalized in some orthogonal basis of representation. Such a basis is unique if none of the diagonal elements are equal, and is a candidate for being the set of pure states that constitute the mixture, according to the IgI. But this interpretation should also allow for a mixture of non-orthogonal pure states, so that there will always be "ignorance" unless the *method of preparation* is known.

4. Preparation procedures

Let us now survey the operational procedures for characterizing the polarization state of a quasi-monochromatic beam of light. Assuming that the photons are not correlated to other systems, then it is sufficient to consider a Hilbert space of dimension K=2 spanned for instance by the vectors $|\phi_0\rangle$ and $|\phi_{90}\rangle$, which correspond to linear polarization at 0° and at 90°, in relation to some reference axis. To measure the polarization state of the beam, the usual procedure is to determine the four "Stokes parameters", which requires measurements of transmittance behind four filters: an isotropic filter, a horizontal linear polarizer (0°), a linear polarizer at 45°, and a right-circularly polarized filter (Shurcliff 1962, 19-25). With these K^2 real numbers one is able to determine the normalized density matrix representing the polarization state of light, which in general will not correspond to a pure state.

To obtain a pure beam, one can simply pass the original mixed beam through a dichroic polarizer, oriented say at 0°. Another way of doing this is to pass the beam through a birefringent analyzer such as a Wollaston prism, which separates the beam

into two orthogonal components, say $|\phi_0\rangle$ and $|\phi_{90}\rangle$ (see figure 1). If a detector is placed in the channel corresponding to $|\phi_{90}\rangle$, the superposition between these two beams will be destroyed with a collapse of the state vector, and one can assume that the undetected beam that has been selected is pure and unentangled. In the 2-dimensional case there is a simple way to check whether the beam is pure, by measuring if all of the beam is transmitted through another appropriately oriented polarizer.

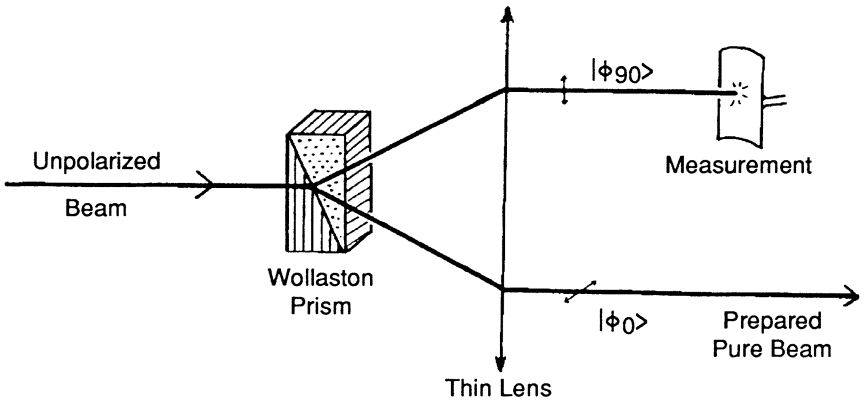

Figure 1: Preparation of a pure beam.

Once pure beams in different polarization states have been obtained in the laboratory, they can be "mixed" with each other, resulting in a beam which is a impure mixture. The simplest way for mixing two light beams is by using a beam splitter such as a half-silvered mirror (see figure 2a), but part of the beams is usually lost. In order to mix two beams with practically no losses, one can reverse the procedure for separating the orthogonal polarization components, as indicated in figure 2b.

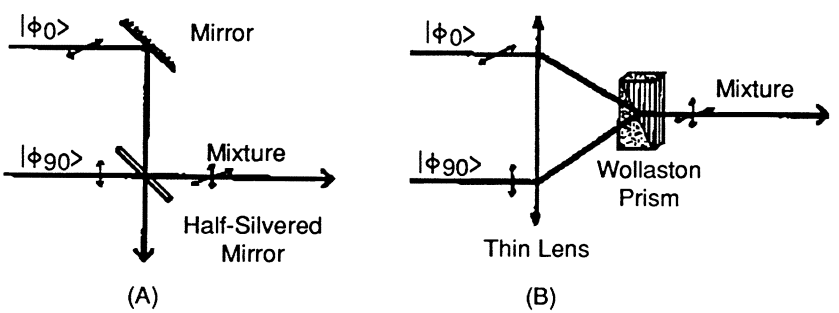

Figure 2: Procedures for mixing pure beams: (a) beam splitter; (b) reversed prism analyzer.

The use of *reversed analyzers* in polarization measurements dates back to Jamin in 1868, with the development of polarization spectroscopy (see Fran(on & Mallick 1971, pp. 55-63). The idea of using reversed analyzers to test the principles of QM was apparently introduced by Bohm (1951, p. 606), with the Stern-Gerlach apparatus, to show that the separation of a pure beam by an analyzer does not destroy the coherence between the beams (state collapse does not occur at the analyzers). The realization of this thought-ex-

periment for single particles has only been achieved in the 80's, for neutron spin (Badurek et al. 1986, pp. 137-141) and for photon phase (Grangier *et al.* 1986, pp. 104-106).

Now that we are able to produce pure beams and to mix them, consider the two following mixtures (Park 1973, pp. 214-5). Mixture A is prepared by combining with equal intensities two pure beams linearly polarized at 0° and at 90°, the states of which are denoted by $|\phi_0\rangle$ and $|\phi_{90}\rangle$. The density operator is given by:

$$\hat{W}_A = (1/2)\cdot |\phi_0\rangle \langle\phi_0| + (1/2)\cdot|\phi_{90}\rangle \langle\phi_0| \qquad (1)$$

Mixture B is prepared by combining equal amounts of pure beams linearly polarized at 45° and at 135°, in states denoted by $|\phi_{45}\rangle$ and $|\phi_{135}\rangle$:

$$\hat{W}_B = (1/2)\cdot |\phi_{45}\rangle \langle\phi_{45}| + (1/2)\cdot|\phi_{135}\rangle \langle\phi_{135}| \qquad (2)$$

These latter states may be expressed as linear combinations of the states polarized at 0° and 90°:

$$\begin{cases} |\phi_{45}\rangle = (1/\sqrt{2})\cdot|\phi_{90}\rangle + (1/\sqrt{2})\cdot|\phi_{90}\rangle \\ |\phi 135\rangle = -(1/\sqrt{2})\cdot|\phi_0\rangle + (1/\sqrt{2})\cdot|\phi_{90}\rangle \end{cases} \qquad (3)$$

Now when we represent $\hat{W}_B$ in terms of the basis $\{\phi_0, \phi_{90}\}$ we obtain the right-hand side of eq.(1). Mixtures A and B are therefore *equivalent*. They are both represented by the density matrix $1/2\cdot\hat{I}$, where $\hat{I}$ is the 2-dimensional identity matrix. This equivalence means that any attempt to distinguish the two mixed beams by means of a polarization analyzer fails. Whatever the orientation θ of the analyzer, the beam intensities measured in both channels would be the same.

Suppose that the analyzer prism is oriented at θ=0° so as to separate any beam into components polarized at 0° and 90° (figure 3). According to the IgI, the photons of mixture A that are in state $|\phi_0\rangle$ will all go through the same channel, falling on the same detector. Likewise for photons in state $|\phi_{90}\rangle$, which go through the other channel. In total, half of the beam will go through each channel.

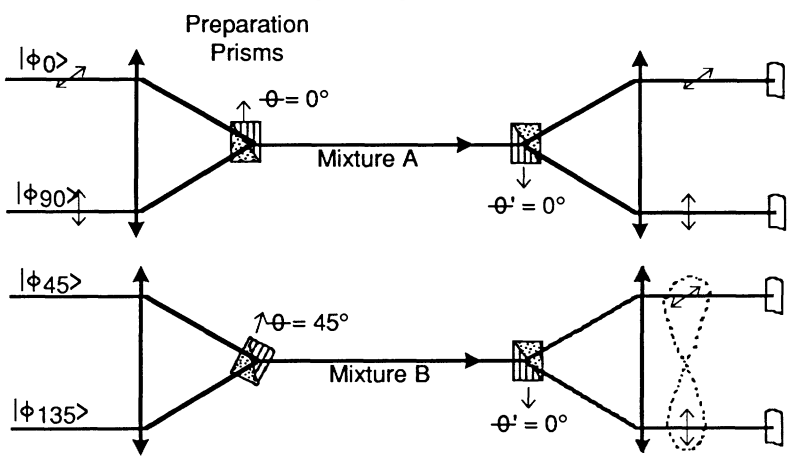

Figure 3: Measurement of polarizations at 0° and 90° on mixtures A and B.

In the case of mixture B, a photon in state $|\phi_{45}\rangle$ has a 50% chance of being counted in each of the two detectors. Likewise for a photon in state $|\phi_{135}\rangle$. In total, roughly half of the photons will be counted in each detector, which is the same as for mixture A. There is however a qualitative difference between the two cases. Before a detection occurs for mixture B, one cannot say that the photon went through one channel or through the other. The system is still in the superposition expressed by eq.(3), represented in figure 3 by dotted ribbons. One can say that the measurement of the polarization component oriented at 0° and 90°, on beam B, involves a *collapse of the state vector*, while on beam A it does not.[3]

5. Fluctuation argument for the ignorance interpretation

An instrumentalist tends to pay more attention to the practical limitations affecting operational procedures than a realist. Notwithstanding this tendency, two different arguments for distinguishing beams A and B have been given by defenders of the IgI, based on the existence of *particle fluctuations*. We will adapt such arguments, given originally for spin-1/2 particles, to the optical example presented in the previous section.

Consider the preparation of mixtures A and B. According to Grossman (1974, 333-338), one can never be sure that *exactly* half of the particles have been prepared in one of the two orthogonal polarization states constituting each mixture. The fluctuations in the preparation of the mixed beams would make them be described by slightly different density matrices, so that they could be distinguished.

The other argument is due to d'Espagnat (1976, pp. 100-102), for the case in which beams A and B have been prepared with *exactly* equal proportions of pure states. Consider again the measurement of linear polarization along 0° and 90° by means of an analyzer (figure 3). For beam A, since exactly half of the photons are in state $|\phi_0\rangle$, and half in state $|\phi_{90}\rangle$, then in the long run there will be no fluctuation in the number of counts obtained on each detector: $\sigma_A = 0$. For beam B, however, each photon has a 50% probability of being detected in each channel, and such random events are subject to fluctuations of the order of $\sigma_B = 1/2 \cdot \sqrt{N}$ (binomial distribution), where N is the mean number of photons per unit time in each beam. Mixtures A and B could therefore be distinguished by their particle fluctuations.

It is curious that these two argumments in favor of the IgI neutralize each other, like parallel coherent waves with opposite phases. D'Espagnat's fluctuations of *measurement outcomes* on the mixed beams is counterbalanced by the fluctuations arising in the *preparation* of the mixtures. In fact, if mixtures A and B are prepared by the procedure indicated in section 4 (figures 1, 2b and 3), and we consider that the initial light source has a Poissonian fluctuation of $\sqrt{N}$ (for a beam of N photons), then the measurements to distinguish beams A and B will yield the *same* fluctuation[4] of $\sigma_A = \sigma_B = (1/2 \cdot \sqrt{N}$.

The fluctuation argument for the IgI breaks down, at least for the preparation setup considered in this paper. The problem remains of whether such an argument can be made valid for some other experimental arrangement, or whether differently prepared but equivalent mixtures can never be distinguished by fluctuation measurements[5].

6. Reversibility as a test for the ignorance interpretation

The instrumentalist and the ignorance interpretations disagree on the issue of whether the process of mixing is *reversible*.

In QM, it is customary to assign an "entropy" to mixtures which is greater than the entropy of pure states (von Neumann 1932, pp. 379-90; Belinfante 1980, pp. 10-5). This accounts for the entropy increase accompaning measurements, since in general a measurement transforms a pure object system into a mixture (a process which involves state collapse). Such a definition of entropy fits in well with the instrumentalist view, for which the procedure of mixing different pure states is always *irreversible.*

For the IgI, however, if one knows how the pure beams were combined, then it is possible to reverse the mixing in a process of "unmixing" which yields the original pure beams again. Such a process would not have to involve any measurement, state collapse, dissipation of energy, or loss of part of the beams. If however one does not know how a mixture was prepared, then one would not know how to reverse the process.

The issue of the reversibility of mixtures can be used as a test in favor of the weak IgI. Consider a "photon cascade" in which pairs of photons correlated with orthogonal polarizations are emitted (see figure 4). The photon cascade used by Aspect *et al.* (1981) is obtained from a beam of calcium excited by tunable laser light, with the pair of correlated photons having frequencies 423 nm and 551 nm. Each pair may be emitted in any direction, but only those heading in opposite directions towards the detectors D_1 and D_2 are selected.

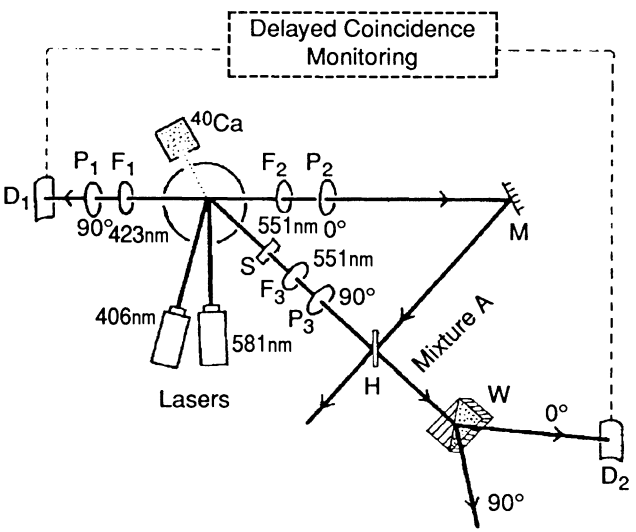

Figure 4: Experiment for reversing the process of mixing.

After appropriate filtration, pairs of photons may be detected in *delayed coincidence.* A photon detected at D_1 "triggers a temporal gate" for detection at D_2, and the probability of a count at D_2 becomes much greater than if no photon had been detected at D_1 (Grangier *et al.* 1986, pp. 101-102). This is a convenient way of "marking" or "individuating" an undetected photon, although in practice one cannot be sure that such a photon will fall upon the gated detector. While the shutter S is closed, the beam passing through analyzer W is pure and oriented at 0°, so that all of the beam falls on D_2 and one can measure a certain coincidence rate (in their setup, Aspect *et*

al. (1981) measured 150 true coincidence counts per second, out of 10^5 individual counts per second at each detector).

Upon opening the shutter, an additional beam polarized at 90° can be mixed to the one polarized at 0° at the beam-splitter H, yielding mixture A defined in eq.(1). If this mixture is then separated by the analyzer into components at 0° and 90_, will the previous coincidence rate still be observed between counts in D_1 and D_2? We would expect so. The mere introduction of the additional beam, which does not modify the beam intensity detected at the final channel at 0°, should not affect the correlations, since different photons do not interact. *We expect the coincidence rate to be maintained* by the mixing and separation of the additional beam, and this would indicate that the *same photons* prepared at 0° from the cascade arrive at the detector after mixing and unmixing. We would have marked one of the two pure subsystems composing the mixture, and been able to retrieve the pure beam after unmixing.

Such a thought-experiment is relatively easy to perform, and if the result we expect turns out to be confirmed, then we have a good argument for claiming that the process of mixing is *reversible*. This lends support to the IgI, since such a view conceives that the pure subsystems composing the mixture maintain their individuality.

7. Conclusion

Two conclusions have been obtained. First, contrary to d'Espagnat's fluctuation argument, it seems that there is no simple way to distinguish between two "different but equivalent mixtures" if the preparation procedure is not known. We have arrived at this conclusion, however, only by looking at a particular type of experimental setup. The problem of whether this result is general or not thus remains open. In particular, it might be possible to distinguish beams described by the same density matrix by means of 2nd order coherence effects·5

The second conclusion is that there is an experimental argument for sustaining that the process of mixing light beams is reversible. In other words, if the preparation is known, we can devise a reversed setup so that we can be confident that the "same" particles constituting the original pure beams will constitute the corresponding unmixed pure beams.

As the photons are detected in D_2 they are not correlated to their pairs anymore, since these had to be previously measured in order to trigger the detection gate. We are therefore assuming that the particles constituting the mixture are not correlated to other systems. Thus, our argument can only lend support to what we have called the weak IgI.

We can now attempt to answer Fano's question. Given an unpolarized beam, we assert that each of the component particles is in a definite polarization state, as long as correlations with other systems in the environment can be neglected. There is however no simple operational means of distinguishing two unpolarized beams. Our adoption of the weak ignorance interpretation is based on those situations in which the procedure for preparing the mixture is known. Implicit in our answer, therefore, is the assumption that beams with known preparation have the same nature as beams the preparation of which we ignore.

Notes

[1]I wish to thank Linda Wessels and Stephen Kellert for discussions on the subject of this paper. Financial support was provided by the "Funda(No de Amparo " Pesquisa do Estado de SNo Paulo" (FAPESP).

[2]To see that such an operator effectively projects the vector $|\Psi\rangle = \sum_k b_k \cdot |\Psi_k\rangle$ onto $|\Psi_j\rangle$, yielding the vector $b_j \cdot |\Psi_j\rangle$, we just need to remember that the inner product $\langle\Psi_j|\Psi_k\rangle$ equals 1 if j=k, and 0 if $j \neq k$ (since the set of eigenvectors is an orthonormal basis of $\mathfrak{H}$). We therefore have: $\hat{P}[\Psi_j]\,|\Psi\rangle = \sum_k b_k \cdot |\Psi_j\rangle\langle\Psi_j|\Psi_k\rangle = b_k \cdot |\Psi_j\rangle$.

[3]The photon is absorbed by an electron during detection, so it becomes ambiguous to speak of a "collapse" in one case but not the other. This ambiguity, however, does not arise for situations such as the detection of particles in a cloud chamber, where only part of the object's energy is absorbed by the detector. Another clear-cut example is that in which the object is an excited atom, and the emitted photon is taken to be the carrier of the interaction between object and measuring apparatus (see Pessoa 1990, pp. 81-91).

[4]Let us write out as $(n \pm \sigma)$ the number of particles and the standard deviation expressing the fluctuations (for reference, see Bevington 1969, pp. 33, 40, 60). For each of the four initial unpolarized beams subject to the Poisson distribution, we have:

$$(n_o \pm \sigma_o) = N \pm \sqrt{N} \tag{4}$$

The selection of a pure beam from each of these mixed beams (figure 1) leads to the beams $|\phi_o\rangle$, $|\phi_{90}\rangle$, $|\phi_{45}\rangle$, and $|\phi_{135}\rangle$ of figure 3, a process involving the binomial fluctuation:

$$(np \pm \sigma) = (1/2)\cdot(N \pm \sqrt{N})\ \ (1/2)\cdot(N \pm \sqrt{N})^{1/2} \approx (1/2)\cdot N \pm (1/\sqrt{2})\cdot\sqrt{N} \tag{5}$$

For mixture A, all of the photons in the pure state $|\phi_o\rangle$ (and only these) go to the upper channel. At detector D_1 we therefore have:

$$(n_A \pm \sigma_A) \approx (1/2)\cdot N \pm (1/\sqrt{2})\cdot\sqrt{N} \tag{6}$$

For mixture B, we can first add the contributions from $|\phi_o\rangle$ and $|\phi_{90}\rangle$:

$$\begin{aligned}(n_m \pm \sigma_m) = &\ [(1/2)\cdot N + (1/2)\cdot N] \pm ([(1/\sqrt{2})\cdot\sqrt{N}]^2 + [(1/\sqrt{2})\cdot\sqrt{N}]^2)^{1/2} \\ = &\ N + \sqrt{N}\end{aligned} \tag{7}$$

This is the same fluctuation as in the source (eq. 4). Now since each photon in mixture B has a 50% chance of being detected in the upper channel, we apply the binomial fluctuation to the whole beam, as in eq.(5):

$$(n_B \pm \sigma_B) = \ (1/2)\cdot(N \pm\sqrt{N}) \pm \ (1/2)\cdot(N \pm \sqrt{N})^{1/2} \approx (1/2)\cdot N\ \ (1/\sqrt{2})\cdot\sqrt{N} \tag{8}$$

Equations (6) and (8) are the same, QED.

[5]It is straightforward to show that the result of endnote 4 can be extended to *any* two equivalent mixtures in *any* finite dimension. This follows as long as the initial unpolarized beams exhibit a Poissonian distribution for photon counts, which is typical of chaotic light sources as well as coherent ones (such as a laser above threshold). In principle, one could distinguish differently prepared but equivalent mixtures for more noisy sources.

The above result assumes that the beam intensities are constant, so that only 1st order coherence effects are present (particle fluctuations). For chaotic sources, this corresponds to sampling times T that are much larger than the "coherence time" τ_0 of the light source (typically 10^{-9} s). If however $T \ll \tau_0$, 2nd order coherence effects become important (wave fluctuations), and the fluctuations are given by $\sigma_0 = (N^2 + N)^{1/2}$ instead of $\sigma_0 = \sqrt{N}$. In this limit, different but equivalent mixtures *can* be distinguished by their fluctuations! An experiment can be readily performed by scattering laser light from plastic balls suspended in water, for which $\tau_0 \approx 10^{-1}$ s (Loudon 1973, pgs. 98-99, 214-221).

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