How to modify the Type of Phenomenon without changing the Quantum-Mechanical State

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A thought-experiment is presented which shows that the "type" of a phenomenon (particle or wave) may be modified without in any way affecting the state of the quantum-mechanical object. The experimental setup consists of a Mach-Zehnder interferometer with polarization devices. The discussion of this apparent paradox centers on the concept of *retrodiction* (defined in terms of conditional probabilities). The overall discussion contrasts different interpretations of quantum mechanics, especially the "complementarity interpretation" and the "wave interpretation".

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1. Complementarity of experimental arrangements. The principle of complementarity is central to the orthodox interpretation of quantum mechanics developed by Niels Bohr (1949). There are, in fact, three different types of "complementarity" in Bohr's writings (see for instance Jammer 1974, 102-4), but the one to be studied here is the complementarity between experimental arrangements (the wave-particle duality). According to this thesis, a quantum mechanical experiment may be represented either in a *corpuscular* picture or in a *wave* picture, never both at the same time. These aspects of experience are *mutually excludent* and, in addition, they are supposed to *exhaust* the description of the atomic object. Leaving aside the question of exhaustion, let us examine what it means for a phenomenon to be corpuscular or undulatory.

2. The Mach-Zehnder interferometer. A "phenomenon", according to Bohr's definition, comprises both the quantum object and the experimental setup, and is only completed when a measurement (a macroscopic registration) takes place. The necessity of this last requirement was illustrated by Wheeler (1978) in his proposal of a "delayed choice experiment" involving a Mach-Zehnder interferometer (Fig. 1).

To understand how this apparatus works, consider classical wave mechanics. A beam of light, represented for simplicity by a one-dimensional monochromatic wave train, is divided into two components at a beam-splitter S_1. The transmitted component follows path A, reflecting off mirror M_1 and falling on beam-splitter S_2. The component reflected to path B travels the same distance and is also split in S_2. It turns out that the amplitudes heading to detector D_2 interfere destructively, while those heading to D_1 superpose constructively. This happens because there is a relative phase shift of a quarter of a wavelength between reflected and transmitted components at each beam-splitter (assumed to be lossless and symmetric). So all of the incoming light ends up in D_1, and nothing is detected in D_2.

This experiment is adequately explained by classical wave mechanics. Quantum theory is only required when the intensity of the incoming beam is greatly reduced and the ordinary detectors are replaced by ones with high sensitivity, such as photomultipliers. One is then capable, upon detection, to discern individual quanta of light (photons). The probability amplitudes for detection in the quantum regime are identical to the electromagnetic wave amplitudes in the classical case. Therefore, all of the quanta fall in D_1 and none in D_2.

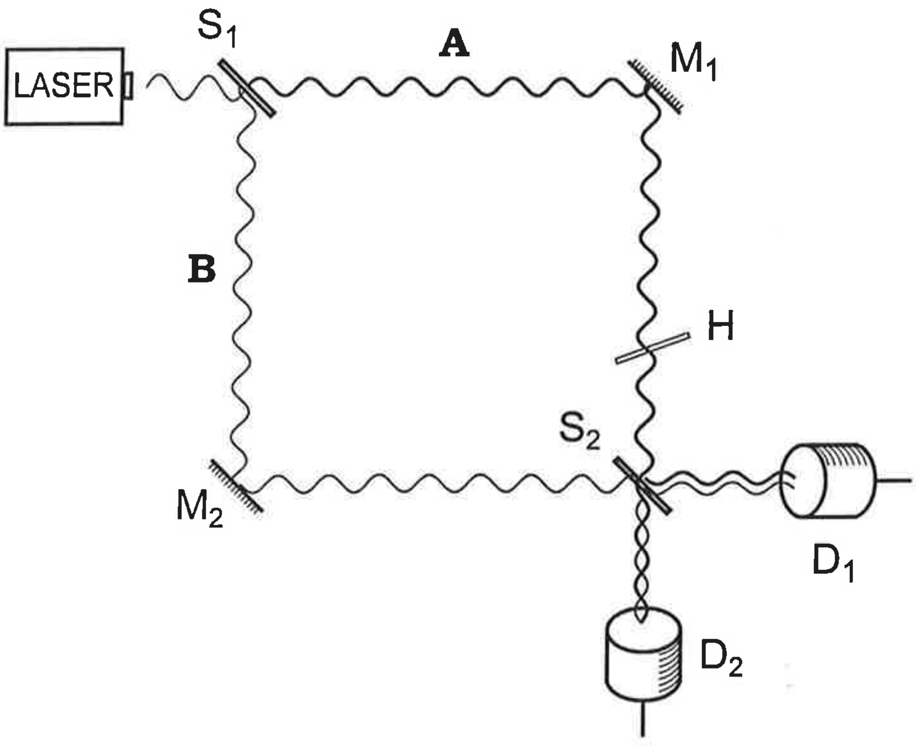

Figure 1: The Mach-Zehnder interferometer.

3. Types of phenomena: wave or particle. The previous quantum-mechanical experiment is a typical "wave phenomenon". One way to justify this name is to argue that the final probability amplitudes can only be explained by assuming the undulatory concepts of constructive and destructive interference. But there is also an operational justification[1] for this expression: if a phase shifter H is placed in path A and the phase shift ϕ of this component is slowly varied in time, the counting rate in each detector will vary in time proportionally to $\cos^2\phi$, a typical interference pattern (see Grangier et al. 1986, Hellmuth et al. 1987).

So the phenomenon is undulatory. But isn't it also corpuscular, since individual quanta can be discerned? No[2]. For a phenomenon to be "corpuscular", one must be able to associate a definite trajectory to the detected quanta. In the experiment of Fig. 1, not only we cannot infer what the trajectory of a specific photon was, but also if we assume that the photon followed, say, path A (nothing going along path B), then there would be a 50% chance of detection in D_2, which is not what happens in the experiment.

Consider what would happen if beam-splitter S_2 were removed, so that no interference would occur. Component A would fall in D_2, while beam B would head to D_1, so that half of the incoming photons would be detected in each photomultiplier (assumed to have perfect efficiencies). In this case, for each photon detected, it seems obvious that one can infer which path was taken by it. In this sense, the trajectories of the detected quanta are known, and this is what characterizes a "corpuscular phenomenon".

The complementarity of experimental setups, therefore, may be stated as the impossibility of having, at the same time, clear interference effects and unambiguous

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[1] There is one basic problem with this operational strategy: the variation of phase also changes the phenomenon! One must therefore stipulate that this kind of modification "preserves the type of phenomenon". For a study of other modifications which preserve the type of phenomenon, see Pessoa (1998).

[2] The fact that detections in the quantum regime involve individual quanta, according to Bohr, is due to Planck's "quantum postulate", not to complementarity. The counterpart of the quantum postulate in the wave interpretation (see footnote 4) is the collapse of the wave packet.

trajectories[3]. This is an empirical claim, so that the study of this wave-particle duality, by itself, does not commit us to any specific interpretation of quantum mechanics.

4. Interpreting the Delayed-Choice Experiment. Wheeler's point, in the delayed-choice experiment, was that the scientist may choose whether the phenomenon will be undulatory or corpuscular (by maintaining or removing S_2) even after the wave packet (associated to the quantum object) has passed by S_1 and enterred the interferometer. This shows that the type of quantum phenomenon (wave or particle) is only established *after* a detection (a macroscopic registration) is completed. Before detection, nothing may be said concerning the nature of the quantum object: in this sense, the complementarity interpretation is "positivist" (quantum theory only describes what is observable). The past is "actualized" by present decisions!

There is, of course, something which is invariant under the removal of S_2: the quantum-mechanical state. If we attribute some sort of reality to the wave function, as is done by the *wave interpretations* of quantum mechanics[4], then the delayed-choice experiment loses its paradoxical character. The delayed decision of the scientist doesn't

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[3] The adjectives "clear" and "unambiguous" were introduced so that the statement would exclude *intermediary phenomena*, which mix wave and particle aspects, but which also come in complementary pairs.

[4] What we have called "wave interpretations" is the class of views which attribute some sort of reality to the wave function $\psi(\boldsymbol{r},t)$ (or to the state vector) or to the probability wave $|\psi(\boldsymbol{r},t)|^2$, without postulating the existence of particles. Within the orthodox camp, the view that a quantum system is represented by a wave function which collapses upon detection (von Neumann) may be classified as a wave interpretation, although no reality is explicitly associated to $\psi(\boldsymbol{r},t)$ (this association, however, might be done privately, when one tries to understand a problem or interpret a calculation). Examples of more realist wave interpretations are the views of Madelung (hydrodynamic view), Schrödinger (who rejected collapses), Everett (who succeded, in a sense, in avoiding collapses with his relative states), GRW(Ghirardi, Rimini & Weber)-Pearle-Gisin (with stochastic collapses), Zeh (decoherence plus relative states), Cramers (retarded and advanced waves), Aharonov-Anandan-Vaidman (protective measurements), and F. Rohrlich (blurred reality).

actualize the past: in this realist view (as in David Bohm's interpretation), the past remains forever the same.

5. Retrodiction. Consider again the corpuscular phenomenon obtained by removing S_2 from the Mach-Zehnder interferometer (Fig. 1). Is it really obvious that a photon detected in D_1 followed the trajectory of path B? If the reader considers it is, then he is implicitly accepting what is known as *retrodiction*, an inference to the past (in this case, of past trajectories). Both Bohr and Heisenberg were aware that retrodiction is an interpretative move, which leads to no contradictions but which is "of a purely speculative character" (Heisenberg, 1930, 20). Yet, retrodiction is implicit in Bohr's complementarity interpretation, in his definition of corpuscular and undulatory phenomena.

How do other interpretations evaluate retrodiction? The ensemble interpretations also tend to accept retrodiction (Ballentine, 1970; Griffiths, 1984). The realist dualism of de Broglie and Bohm rejects its usual form (trajectories may be inferred after detection, but they are different from the straight paths of orthodox retrodiction). The wave interpretation mentioned in the previous section emphatically rejects any ontological import for retrodiction, although it may accept that the formal definition of retrodiction to be mentioned in section 9 is useful for classifying different experimental situations.

6. The polarization interferometer. We could now modify the experimental setup by introducing mutually orthogonal polarizers in the arms of the Mach-Zehnder interferometer. But instead of using polarizers, which absorb part of the beam, suppose that the initial beam is 0° linearly polarized, and that a $\pi/2$ polarization rotator *R* is introduced in path *B*, changing the polarization in this arm from 0° to 90° (Fig. 2). In addition, each detector of Fig. 1 is replaced by a polarization analyzer (a birefringent prism), which separates any incoming beam into components polarized at 0° and at 90°, each of which falls on a separate detector.

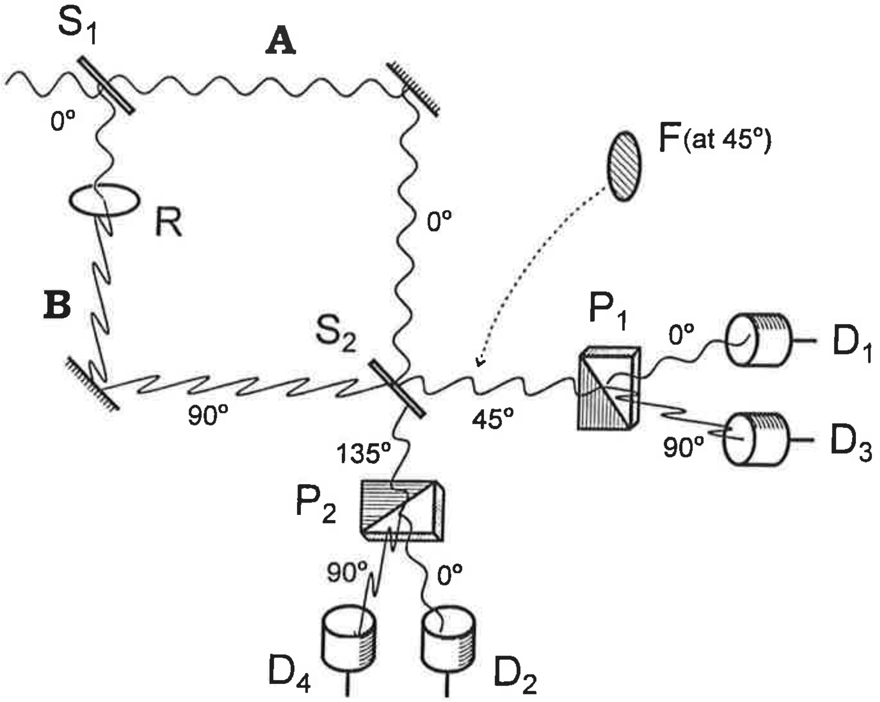

Figure 2: Mach-Zehnder interferometer with polarization devices.

In the regime of moderate beam intensities and ordinary detectors, classical wave mechanics pictures the component at *A* as a transversally oscillating electromagnetic wave, vibrating along a specified direction **x** (0°), while that at *B* as a transverse wave oscillating along the orthogonal **y** direction (90°). After recombination at beam-splitter S_2, the orthogonal components cannot interfere destructively or constructively as in the setup of Fig. 1. Linear superposition of the waves oscillating along the **x** and **y** axes will occur, but the resulting wave will be polarized at 45° (in the case of the component heading towards prism P_1) or at 135° (for the component going to analyzer P_2; the difference, once again, is due to the relative phase shift between reflected and transmitted beams). In the classical regime, the beams will be separated by each polarization analyzer into equal parts, and all of the detectors in Fig. 2 will register the same intensity of light.

What happens in the quantum regime? A single photon which enters the interferometer will have equal probabilities of being measured in each of the detectors. If the photon is detected in D_1, that means it was detected with 0° polarization. But in this case, it must have followed path A, which which is associated to 0° polarization. So the trajectories are known: the phenomenon is corpuscular! The principle of complementarity may be checked (see section 3) by placing a phase shifter in path A and varying ϕ: no variation in the detection counts will be measured, so the phenomenon is not undulatory.

7. Interpreting the polarization interferometer. In this experiment, according to the complementarity interpretation, one is allowed to picture the detected photon as a particle which followed a well-determined path. If the particle passed through the rotator, its polarization state was changed; if not, it remained the same: this "information" is carried along with the particle until it is detected.

But are we allowed to infer the photon's trajectory? Didn't we agree in the previous section that, according to classical wave mechanics, the wave propagating between S_2 and

P_1 is linearly polarized at 45°? If that's the case, how can this wave at 45° have a "memory" of what its original components were?

According to the wave interpretation of quantum mechanics, this memory *is* lost, and one cannot claim that the photon detected in D_1 came through path A. One cannot accept retrodiction, according to this view. However, the acceptance of retrodiction is tenable: if one adopts the complementarity interpretation or a more corpuscular view such as the ensemble interpretations tend to be, then the assumption of retrodiction leads to no inconsistencies. One may, in fact, give interesting "plausibility arguments" in favor of retrodiction (see for instance Vaidman et al., 1987), but these will not be examined here.

8. The Paradox: Modifying the Type of Phenomenon without Changing the State.

Consider now a polarization filter *F*, which transmits completely a beam of light linearly polarized at 45°, and blocks completely a beam of light polarized at 135°. Suppose that *F* is inserted between S_2 and P_1, as shown in Fig. 2. What will happen?

The orientation of the polarizer is chosen so that it leaves unchanged the beam of light between S_2 and P_1. In other words, the beam is in the eigenstate of the projection operator (with eigenvalue 1) associated to the filter. The result of this choice is that the quantum-mechanical state of the system is not modified by the insertion of the filter, and no change occurs in the probabilities of measurement in each of the detectors.

Yet, the type of phenomenon associated to detector D_1, which was previously corpuscular, now becomes undulatory! To check this, we may place a phase shifter in path A and observe that a variation in ϕ in fact leads to a $\cos^2\phi$ variation in the counting rates at

D_1 and D_3[5]. The filter that has been inserted acts as a "quantum eraser" (see Herzog et al., 1995); one may say (rather imprecisely) that information about the path has been removed by the filter.

Notice that the variation in ϕ mentioned above does not change detection rates in D_2 and D_4. The phenomenon associated to these detectors is still corpuscular, while that associated to D_1 and D_3 is undulatory. This shows that the concept of "phenomenon" refers to the individual quantum that has been detected.

9. The Definition of Retrodiction. How can a modification of the apparatus, that does not affect the state of the quantum object, modify the type of phenomenon? This might not seem surprising if we remember Bohr's assertion that a phenomenon should be defined for the object *plus* the apparatus, so that if the apparatus is modified, so is the phenomenon[6]. But how do we know whether a modification of the apparatus will change the type of phenomenon or not?

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[5] It is true that when ϕ is changed and made different from zero, part of the light beam is absorbed by the filter, changing the object's state. But this change in ϕ is only an operational test for checking whether the phenomenon with $\phi = 0$ is undulatory. The fact remains that, when $\phi = 0$, the phenomenon is undulatory and the state is the same as if the filter had not been inserted.

[6] He arrived at this conclusion in order to explain the EPR paradox. Consider Bohm's EPR setup involving the measurement of the spins of a pair of correlated particles (Bohm 1951, 611-23). A Stern-Gerlach magnet is used to separate orthogonal states of a particle before detection. Different orientations of the magnet correspond to different observables being measured. However, in this example, a change in orientation of a magnet also *changes the state of the object* that passes through it, contrary to what happens in the experiment examined in this paper.

To characterize a phenomenon one must first define *retrodiction*. There are at least two different ways of doing this, one of which is in terms of conditional probabilities[7]. Consider the interferometer of Fig. 2 without filter F, and let us focus on a quantum registered in detector D_1. If a photon follows for sure path A (say, by removing S_1), it has probability ½ of falling in D_1: Prob(D_1/A) = ½. If it follows for sure path B (after replacing S_1 by a mirror), then it has probability 0 of falling on D_1: Prob(D_1/B) = 0. So, in this case, given that it was registered in D_1, we infer that the probability that it went through path A is 1 (it couldn't have gone through path B and fallen in D_1): Prob(A/D_1) = 1. Since we are able to infer[8] a definite trajectory, the phenomenon is corpuscular.

Now consider the setup of Fig. 2 with filter F inserted. If the photon takes for sure path A (with S_1 removed) it will be 0° polarized: there will then be probability ½ of reflection at S_2, ½ of not being absorbed at the filters, and ½ of taking the path from P_1 to D_1. The final probability of detection in D_1 is therefore 1/8: Prob(D_1/A) = 1/8. If the photon surely follows path B (replacing S_1 by a mirror), the probability is the same: Prob(D_1/B) =

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[7] The other type of definition is in terms of the inverse unitary evolution operator. For a quantum detected in D_1, take the eigenstate $|D_1\rangle$ associated to the detection (at time t_3 right before detection) and apply the inverse unitary evolution $\hat{U}^{-1}(t_3, t_1)$ to time t_1 when the quantum was inside the interferometer. If $|A\rangle$ is the eigenstate associated to path A (with the appropriate polarization state) and $|B\rangle$ to path B, then one associates the following state (after normalization) to the detection at D_1: $\langle A|\hat{U}^{-1}|D_1\rangle\,|A\rangle + \langle B|\hat{U}^{-1}|D_1\rangle\,|B\rangle$. This defines a basis state of the orthonormal set which defines the *phenomenon*. The set which defines the corpuscular phenomenon is $\{|A\rangle, |B\rangle\}$, while an undulatory phenomenon is defined by $\left\{\frac{1}{\sqrt{2}}(|A\rangle + |B\rangle), \frac{1}{\sqrt{2}}(|A\rangle - |B\rangle)\right\}$. *Complementary phenomena* are those associated to so-called "mutually unbiased bases". This definition is applicable to intermediary phenomena (see Pessoa 1998).

[8] One could have used Bayes' rule, considering that Prob(A) = ½ and Prob(D_1) = ¼. The problem, of course, with the preceding inference is that Prob(D_1/A) has been calculated for one phenomenon (S_1 removed), Prob(D_1/A) for another (S_1 replaced by a mirror), and Prob(A/D_1), Prob(A) and Prob(D_1) for yet another (S_1 in place). The assumption that the inference is valid, in spite of referring to different experimental setups, is what makes retrodiction an interpretative supposition.

1/8. So, in this case, detection at D_1 leads to an inference[9] of equal probabilities of having followed path A or path B: the phenomenon is undulatory.

But how can the insertion of filter F change the type of phenomenon? It is true that it doesn't affect the state of the quantum object, but there is a probability of absorption by the filter when the photon is surely in path A, or surely in path B. Retrodiction is an interpretative assumption which, in a certain sense, privileges the decomposition of the state vector into these bases which evolved from well-localized states. Since these components, when alone, are partially absorbed by the filter (although their sum is not), and since they are implicitly used in the definition of phenomena, it follows that the insertion of the filter may change the phenomenon, even though the quantum state (the sum of the components) is not changed.

11. Conclusions. The thought-experiment explored in this paper shows clearly how the wave-particle duality (complementarity of experimental arrangements) is intimately connected with retrodiction. The explanation given to the paradox shows that the type of phenomenon is not defined solely by the object's state evolution or by the counting rates at the detectors. It is in agreement with Bohr's idea that the concept of "phenomenon" is *relational*, involving the relation between object and apparatus.

On the other hand, interpretations that attribute ontological significance to the state vector might take this thought-experiment as an argument *against* the plausibility of

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[9] In this case, Bayes' rule does not work in a straightforward way: one obtains Prob(A/D_1)= Prob(B/D_1)= ¼. To understand why this happens, one may consider the alternative way of defining retrodiction (see note 7). As the eigenstate associated to D_1 evolves back in time and falls on filter F, part of the beam is lost (to maintain unitarity, replace the filter by another polarization analyzer). If the probabilities of these lost components are added to Prob(A/D_1) and Prob(B/D_1), one obtains 1.

retrodiction. But even if this view is adopted, one must acknowledge the explanatory power of the principle of complementarity: in one case (Fig. 2 without filter *F*), a variation of phase shift leads to an interference pattern, while in the other (Fig. 2 with *F*) it does not.

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